How delay equations arise in Engineering?

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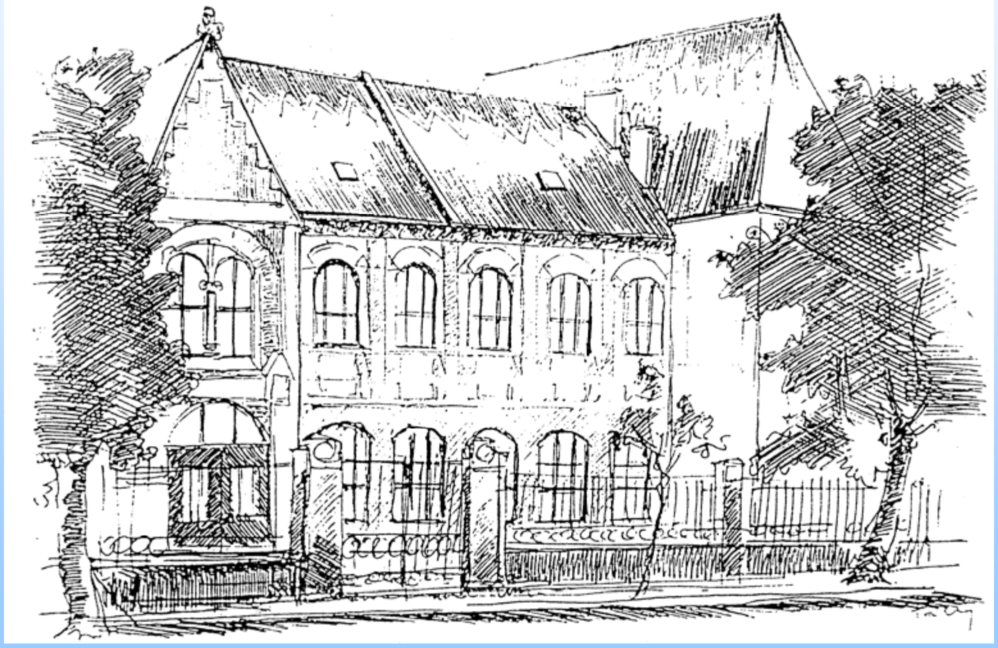


Contents

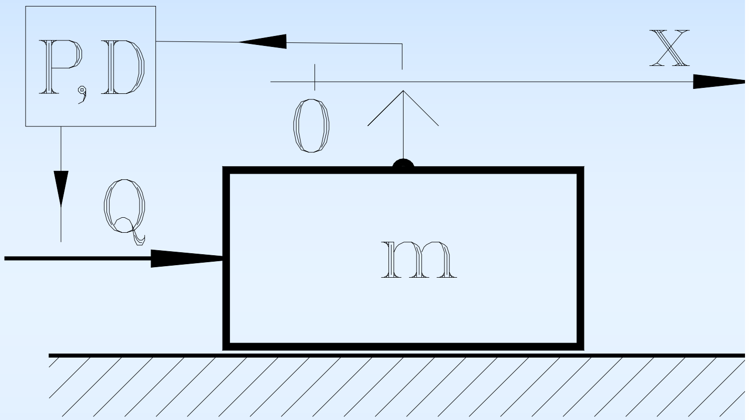
Answer: Delay equations arise in Engineering…

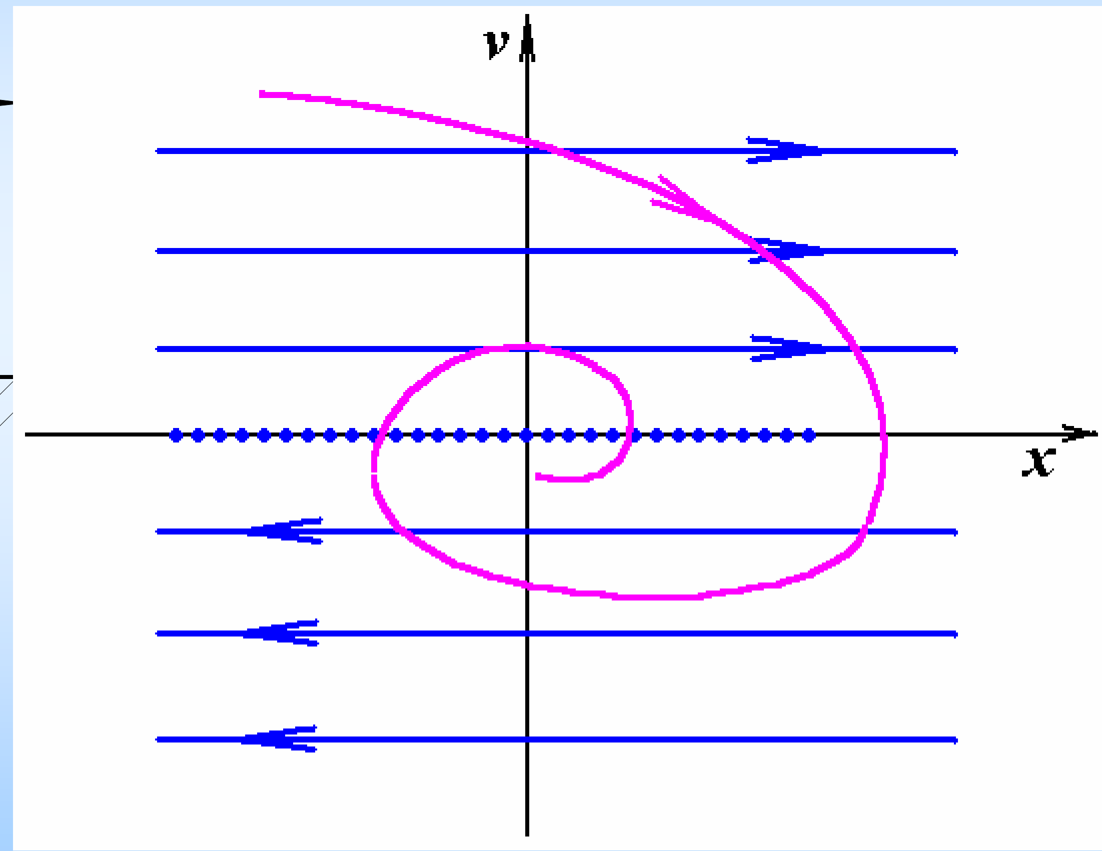
… by the information system (of control), and
by the contact of bodies.

- Linear stability & subcritical Hopf bifurcations
- **Robotic position and force control**
- Balancing – human and robotic
- Contact problems
- Shimmying wheels (of trucks and motorcycles)
- Machine tool vibrations

Position control

1 DoF models $\Rightarrow$ x





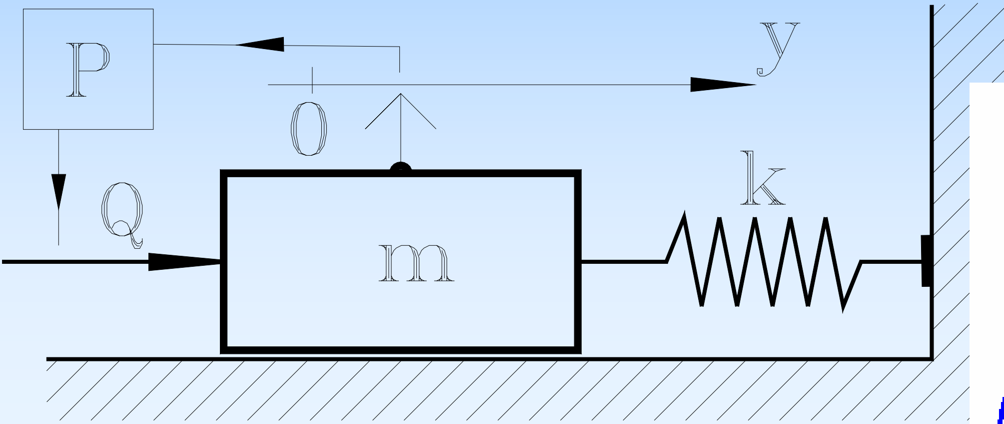
Blue trajectories:

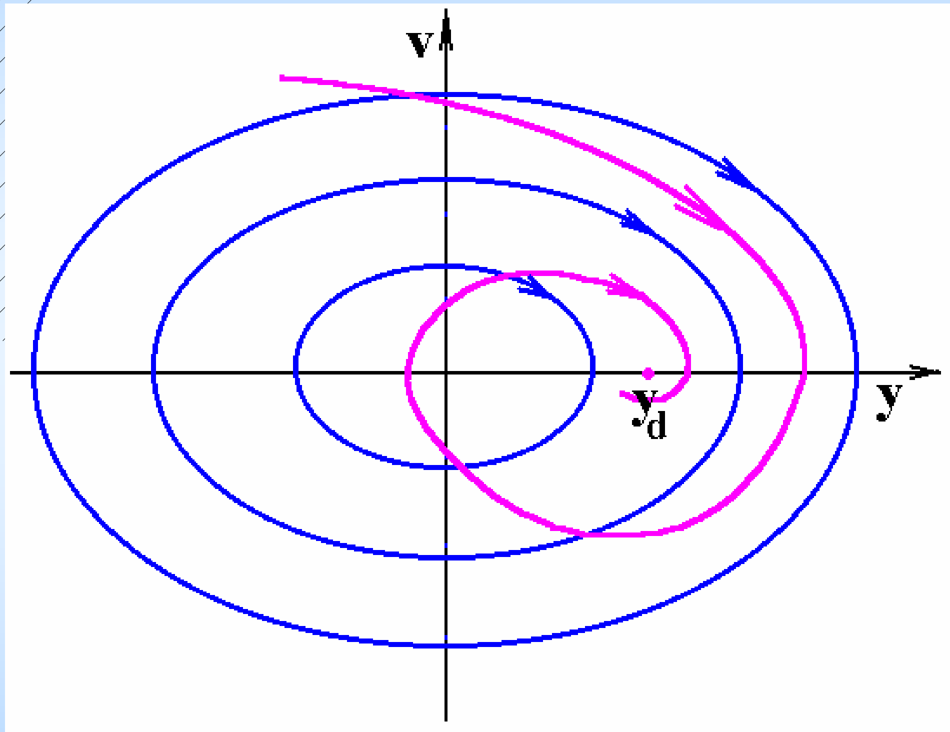
$$Q = 0$$

Pink trajectories:

$$Q = -Px - D\dot{x}$$

Force control





Desired contact force:

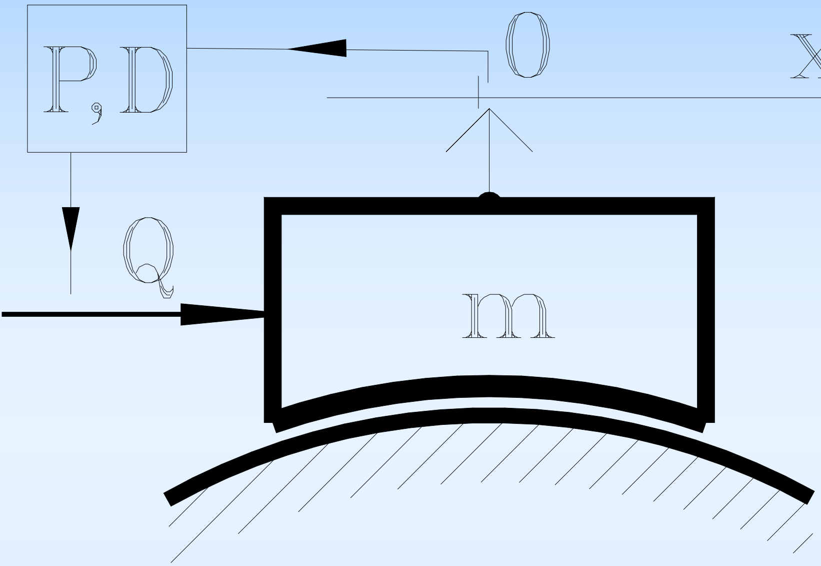
$F_d = k y_d$;

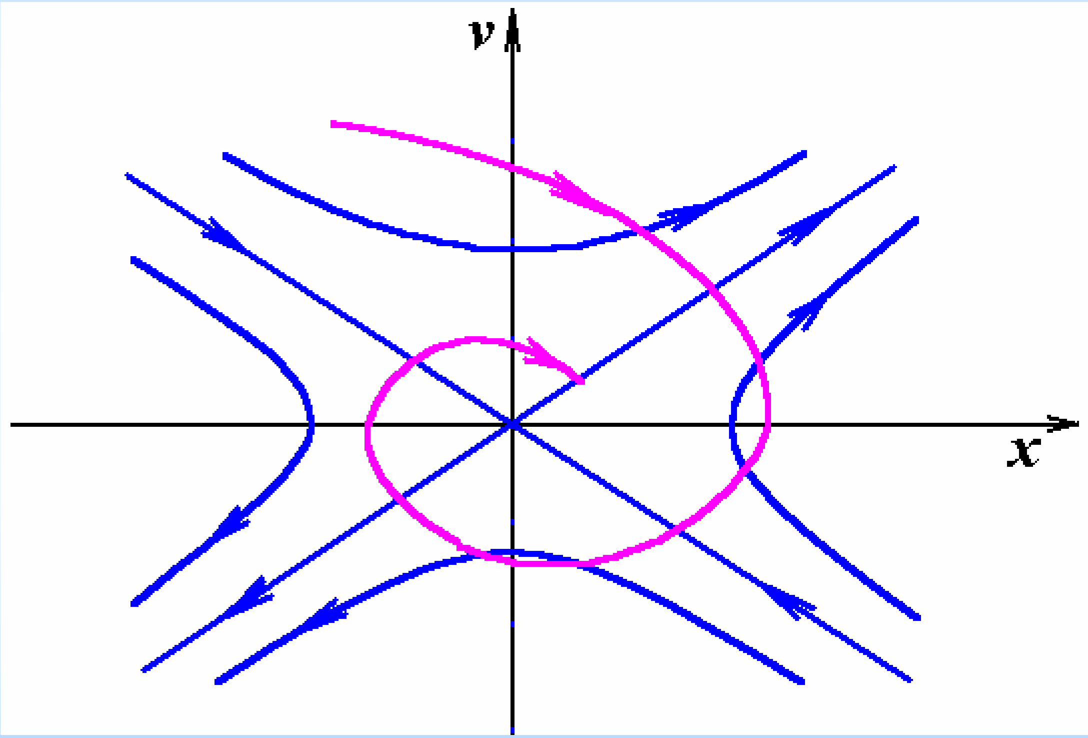
Sensed force:

$F_s = k y$

Control force: $Q = -P(F_d - F_s) - D\dot{F}_s + F_{s \text{ or } d}$

Stabilization (balancing)





Control force:

$$Q = -Px - D\dot{x}$$

Special case of force control: with $k < 0$

Modeling digital control

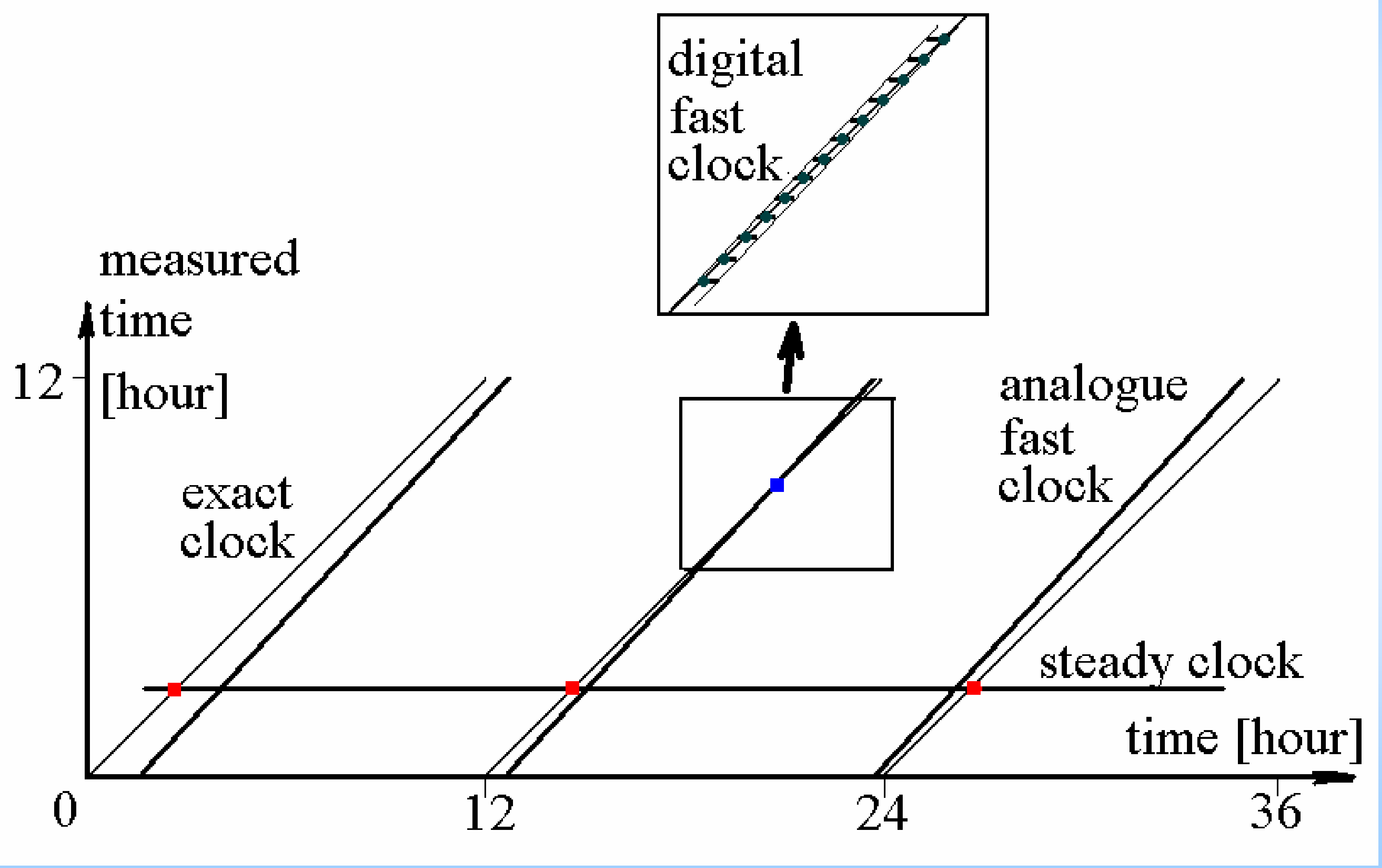
Special cases of force control:

- position control with zero stiffness ($k = 0$)
- stabilization with negative stiffness ($k < 0$)

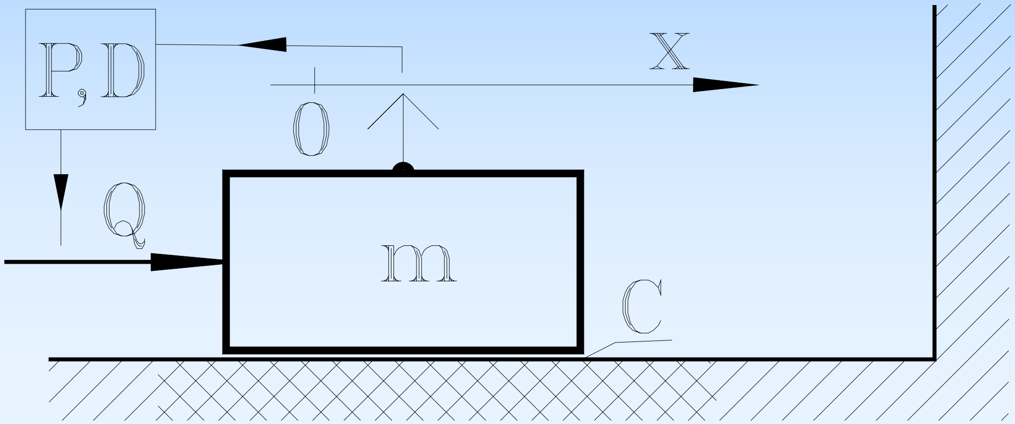
Digital effects:

- quantization in time: *sampling* – **linear**
- quantization in space: *round-off* errors
  at ADA converters
  – **non-linear**

Alice's Adventures in Wonderland



Digital position control



Equation of motion

$$m\ddot{x}(t) + D\dot{x}(t) + Px(t) = -C\,\mathrm{sgn}\,\dot{x}(t)$$

Position error: $\Delta = C / P$

Stability $\Leftrightarrow$ $P > 0,\ D > 0$

Modeling sampling

Time delay τ and zero-order-holder

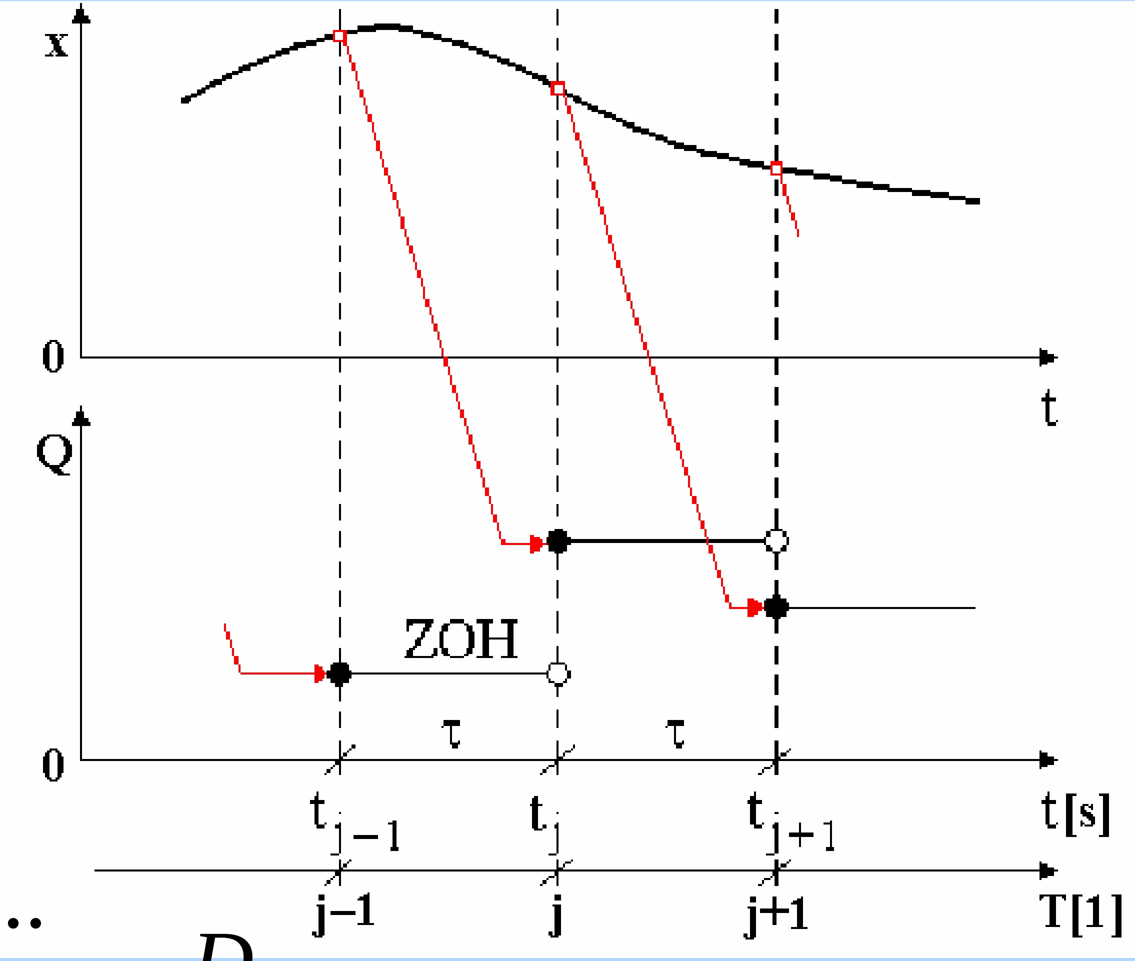
Dimensionless time

$$T = t/\tau$$

Equation of motion

$$\frac{m}{\tau^2} x''(T) = Q(T)$$

where for $j = 1,2,\ldots$



$$Q(T) \equiv -Px(j-1) - \frac{D}{\tau} x'(j-1),\ T \in [j, j+1)$$

Stability of digital position control

$$x''(T) \equiv \underbrace{-px(j-1) - dx'(j-1)}_{=:\, a_j}, \quad T \in [j, j+1)$$

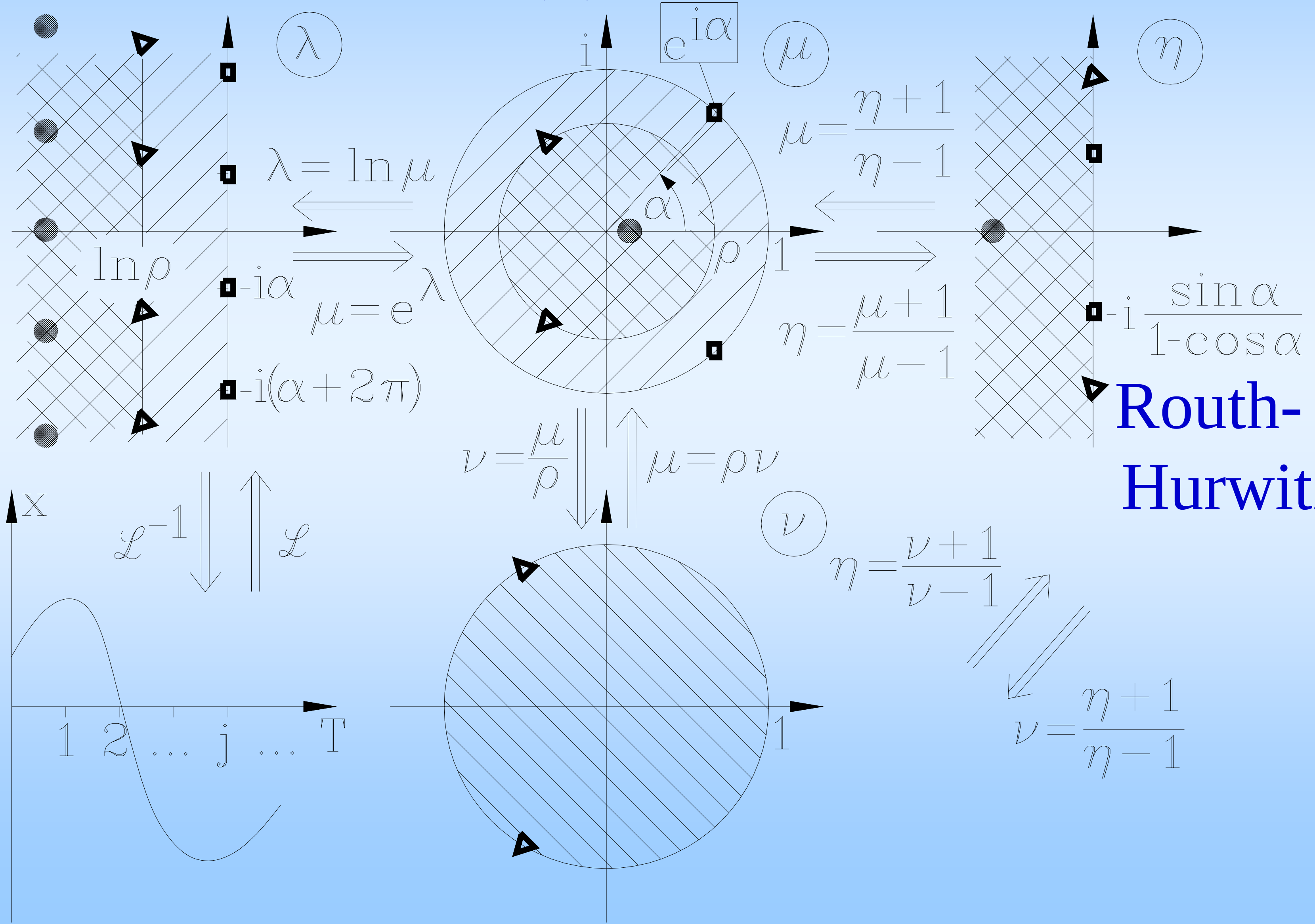
$$x(T) = x(j) + x'(j)(T-j) + \tfrac{1}{2} a_j (T-j)^2$$

$$x'(T) = x'(j) + a_j (T-j), \quad T \in [j, j+1)$$

$$\mathbf{z}^j := \begin{pmatrix} x(j) \\ x'(j) \\ a_j \end{pmatrix} \Rightarrow \mathbf{z}^{j+1} = \mathbf{A}\mathbf{z}^j, \quad \mathbf{A} = \begin{pmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 1 & 1 \\ -p & -d & 0 \end{pmatrix}$$

$$\det(\mu \mathbf{I} - \mathbf{A}) = \mu^3 - 2\mu^2 + (1 + d + \tfrac{1}{2} p)\mu + (\tfrac{1}{2} p - d) = 0$$

Checking $|\mu_{1,2,3}|<1$ algebraically



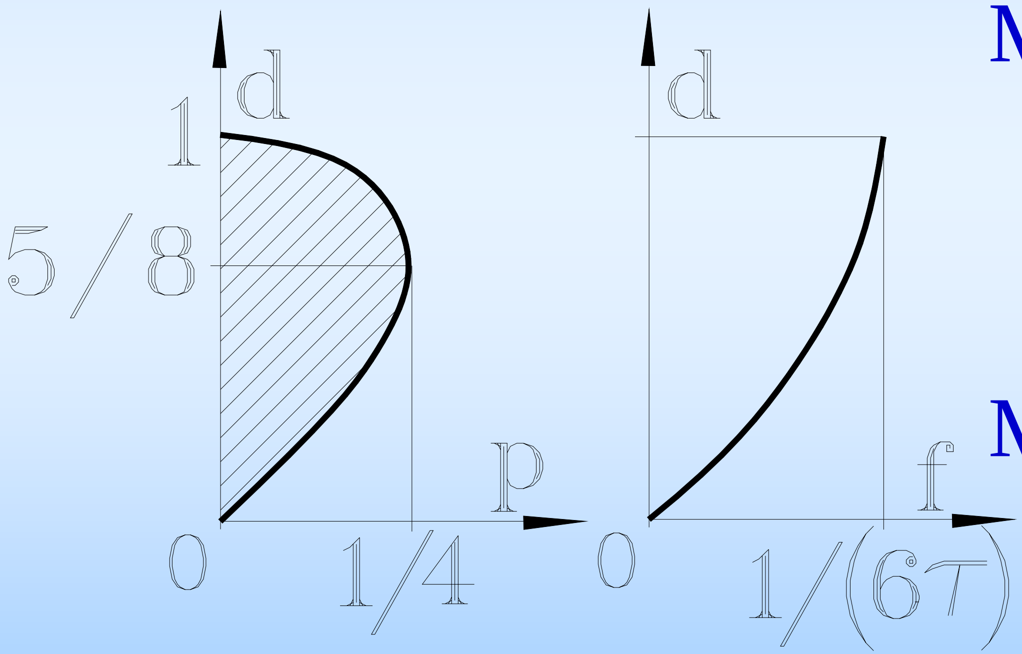
Routh-Hurwitz

Stability chart

$$\operatorname{Re}\eta_{1,2,3} < 0$$

$$p\eta^3 + 2(d-p)\eta^2 + (4-4d+p)\eta + 2(2+d) = 0$$

Stability conditions: $p > 0$, $H_2 > 0$ ($= 0 \Rightarrow$ Hopf)



Maximum gain:

$$P_{max} = \frac{1}{4}\frac{m}{\tau^2}$$

Minimum position error

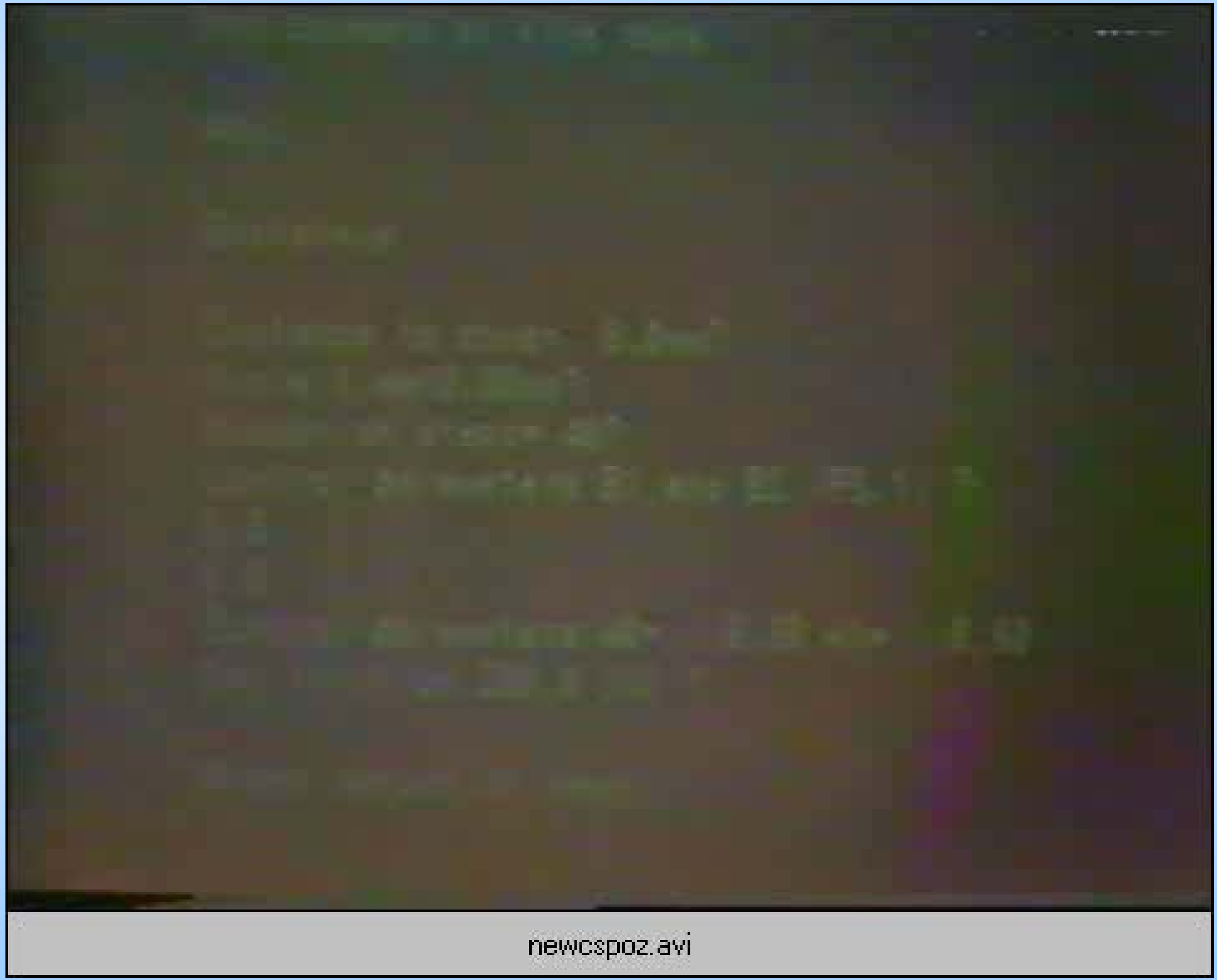
$$\Delta_{min} \geq 4C\tau^2 / m$$

Self-excited vibration frequency: $0 < f < f_{sampling}/6$

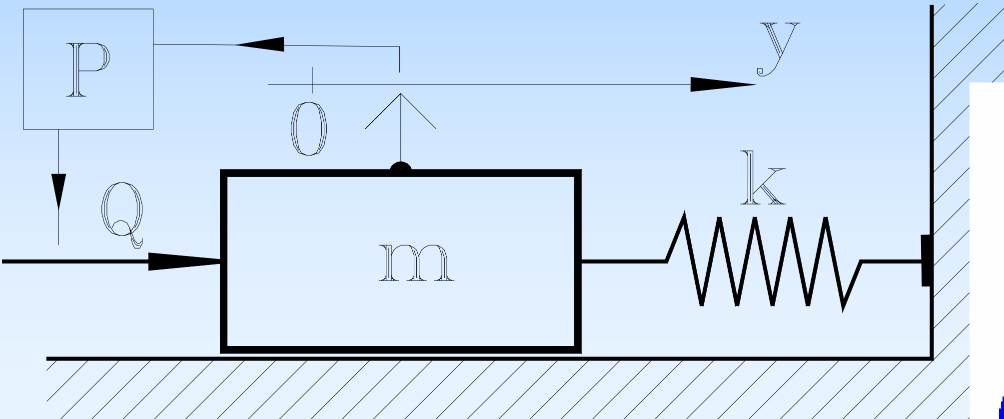
The low-frequency vibrations

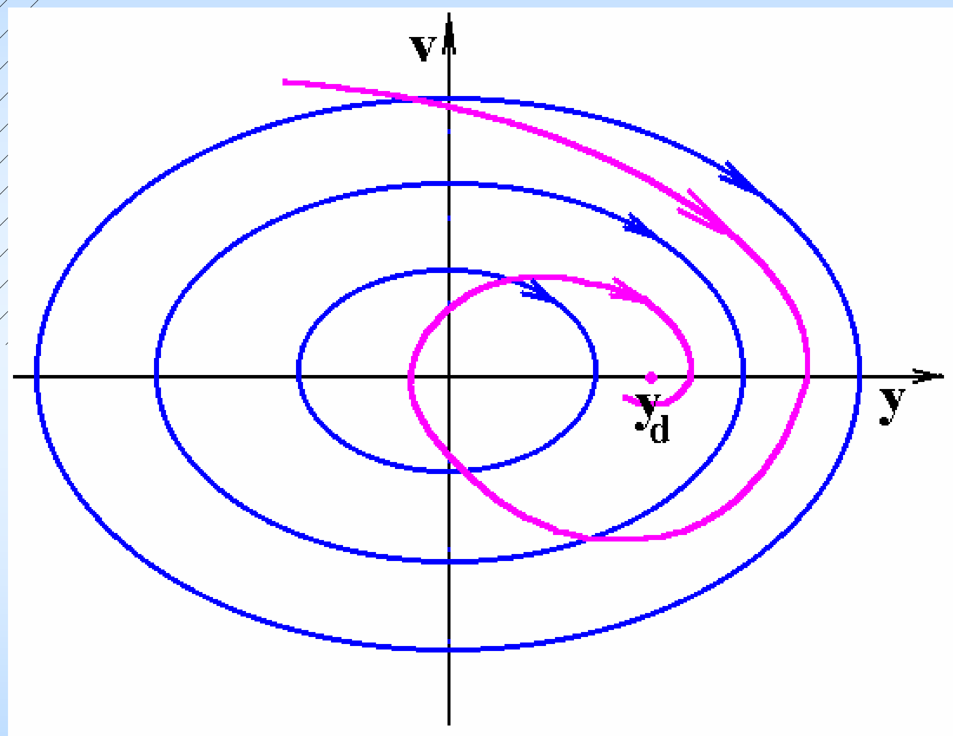
prob_poz.avi

The low-frequency vibrations



Force control





Desired contact force:

$$F_d = ky_d\ ;$$

Sensed force:

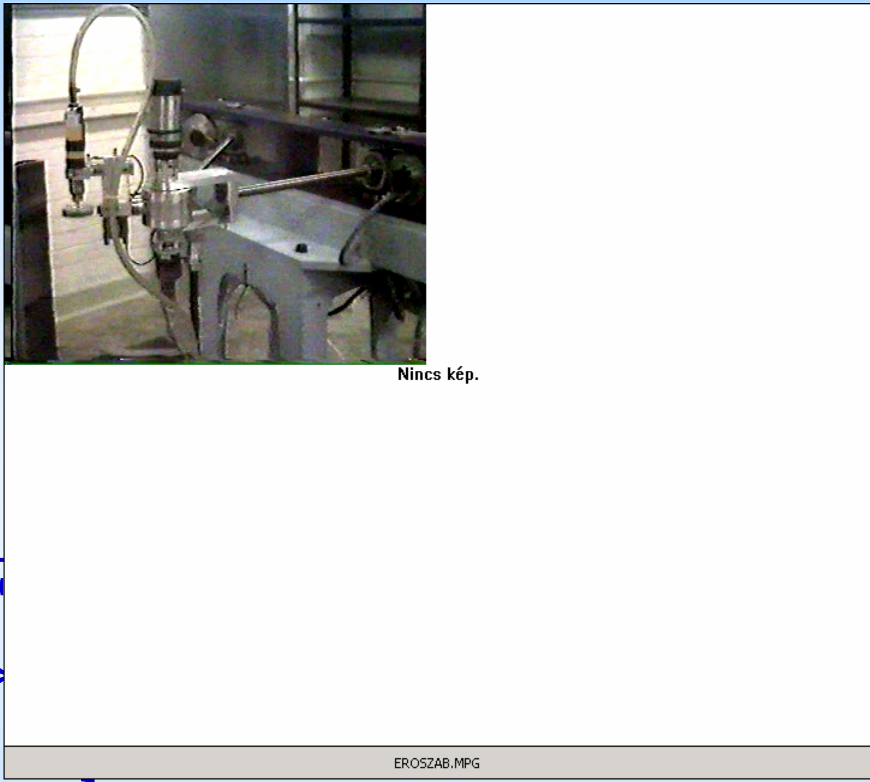
$$F_s = ky$$

Control force: $Q = -P(F_d - F_s) - D\dot{F}_s + F_{s \text{ or } d}$

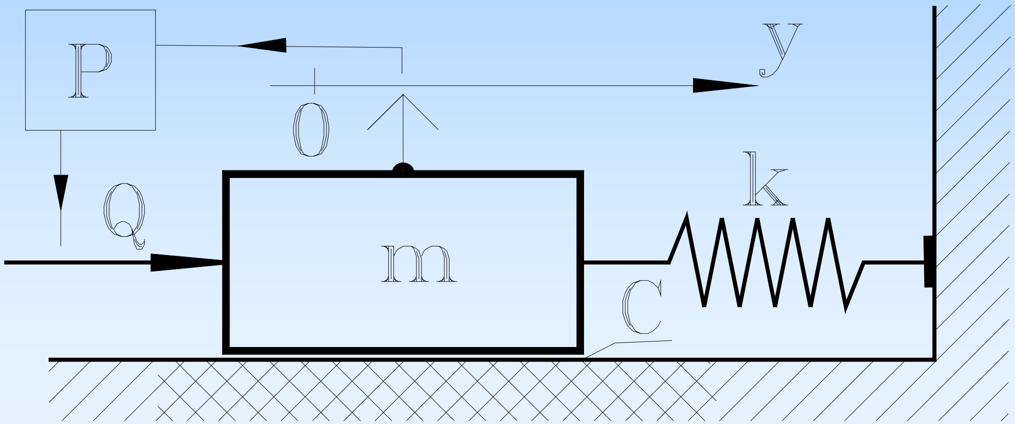
Force control – motivation

- Polishing turbine blade ⇨
  (Newcastle/Parsons robot)
- Rehabilitation robotics
  (human/machine contact)
- Coupling force control (CF
  (between truck and trailer)*
- Electronic brake force control
  (added to ABS systems)*

  * ©Knorr-Bremse



Digital force control



Equation of motion:

$$m\ddot{y}(t) + ky(t) = -P(ky(t) - F_d) + \left\{ \begin{matrix} ky(t) \\ \text{or} \\ F_d \end{matrix} \right\} - C\,\mathrm{sgn}\,\dot{y}(t)$$

Equilibrium: $y_d = F_d / k$

Force error: $\Delta_F = C / P$ or $C/(1+P)$ (Craig '86)

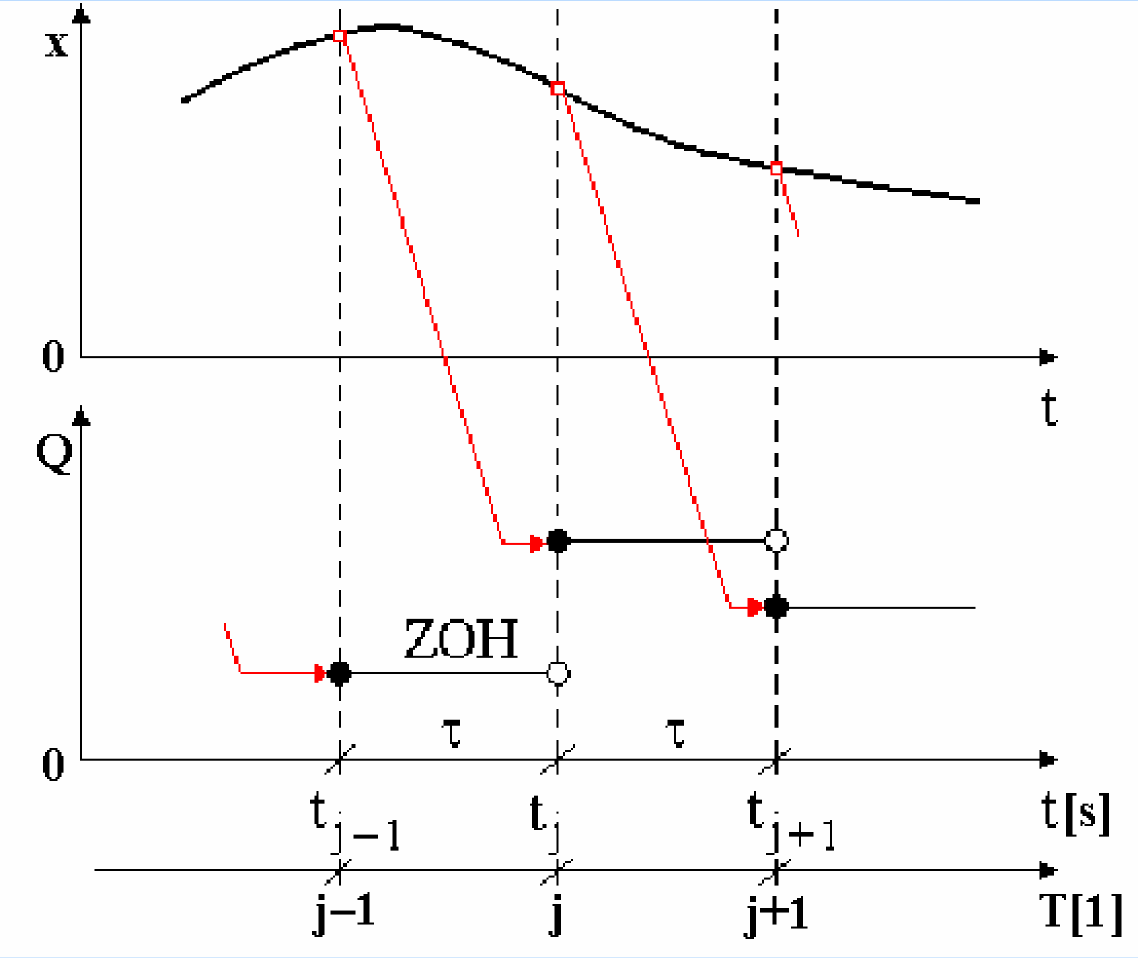
Stability for $y(t) = y_d + x(t)$, $m\ddot{x} + Pkx = 0 \Rightarrow P > 0$

Modeling sampling

Time delay τ and zero-order-holder (ZOH)

Dimensionless time

$$T = t / \tau$$



Modeling sampling

Sampling time is τ, the j^{th} sampling instant is $t_j = j\tau$

$$Q(t) \equiv -P\Big(ky(t_j - \tau) - F_d\Big) + ky(t_j - \tau),\, t \in [t_j, t_j + \tau)$$

Natural frequency: $f_n = \omega_n/(2\pi) = \sqrt{k/m}/(2\pi)$

Sampling frequency: $f_s = 1/\tau$ time: $T = t/\tau$

Dimensionless equations of motion: $T \in [j, j+1)$

$$x''(T) + (\omega_n\tau)^2 x(T) = (\omega_n\tau)^2 (1-P)x(j-1)$$

$$x(T) = x_h(T) + x_p(T) =$$ $x(j), x'(j) \Rightarrow B_1, B_2$

$$B_1 \cos(\omega_n t) + B_2 \sin(\omega_n t) + (1-P)x(j-1)$$

Stability of digital force control

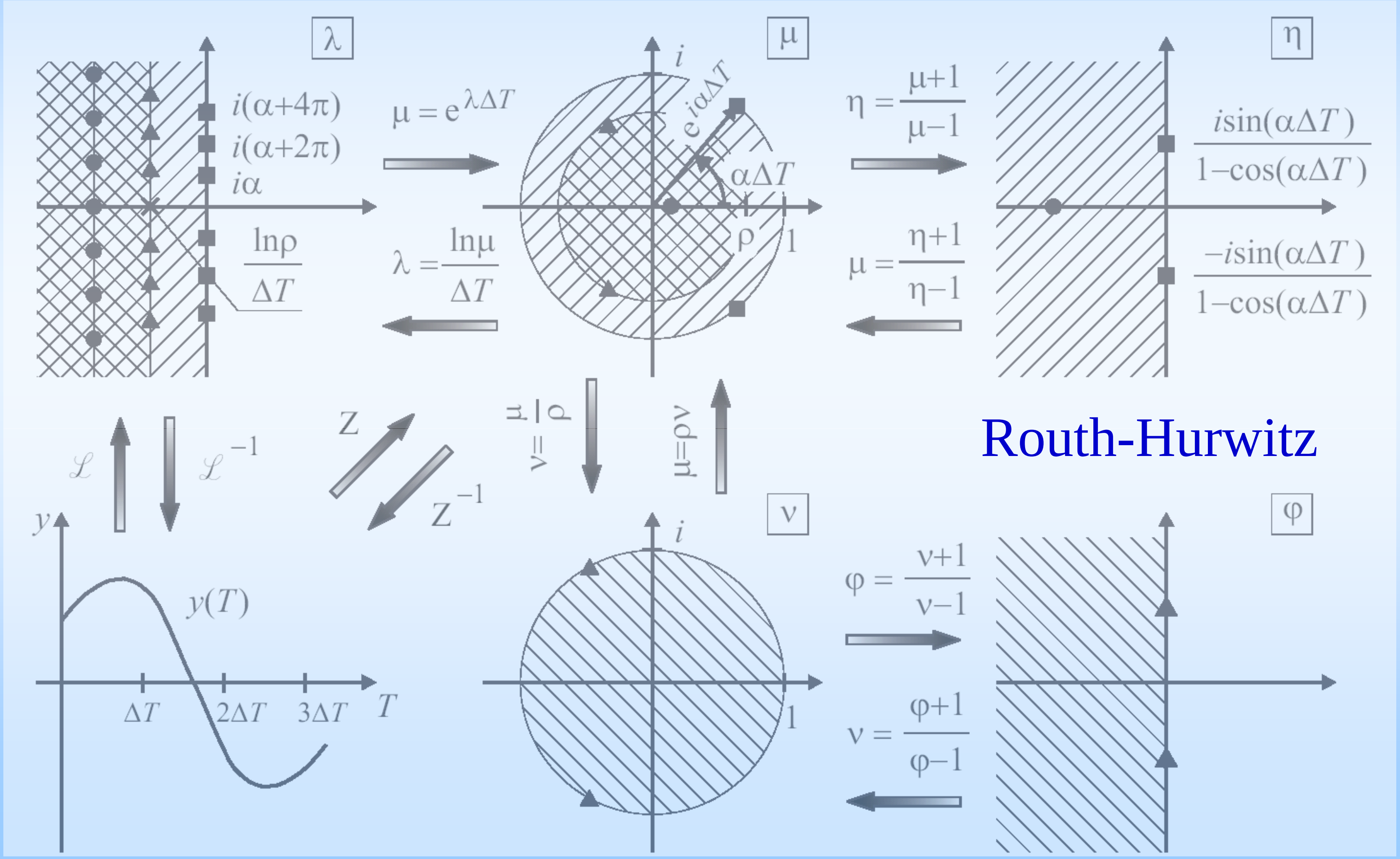
$$\mathbf{z}^j = \begin{pmatrix} x(j-1) \\ x(j) \\ x'(j) \end{pmatrix} \Rightarrow \mathbf{z}^{j+1} = \mathbf{A}\mathbf{z}^j \Rightarrow \det(\mu\mathbf{I} - \mathbf{A}) = 0$$

$$|\mu_{1,2,3}| < 1 \Leftrightarrow \text{stability}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ (1-P)(1-\cos(\omega_n\tau)) & \cos(\omega_n\tau) & \frac{1}{\omega_n\tau}\sin(\omega_n\tau) \\ (1-P)\omega_n\tau\sin(\omega_n\tau) & -\omega_n\tau\sin(\omega_n\tau) & \cos(\omega_n\tau) \end{pmatrix}$$

Parameters: $(\omega_n\tau)/(2\pi) = f_n / f_s$ and P

Checking $|\mu_{1,2,3}|<1$ algebraically



Routh-Hurwitz

Stability chart of force control

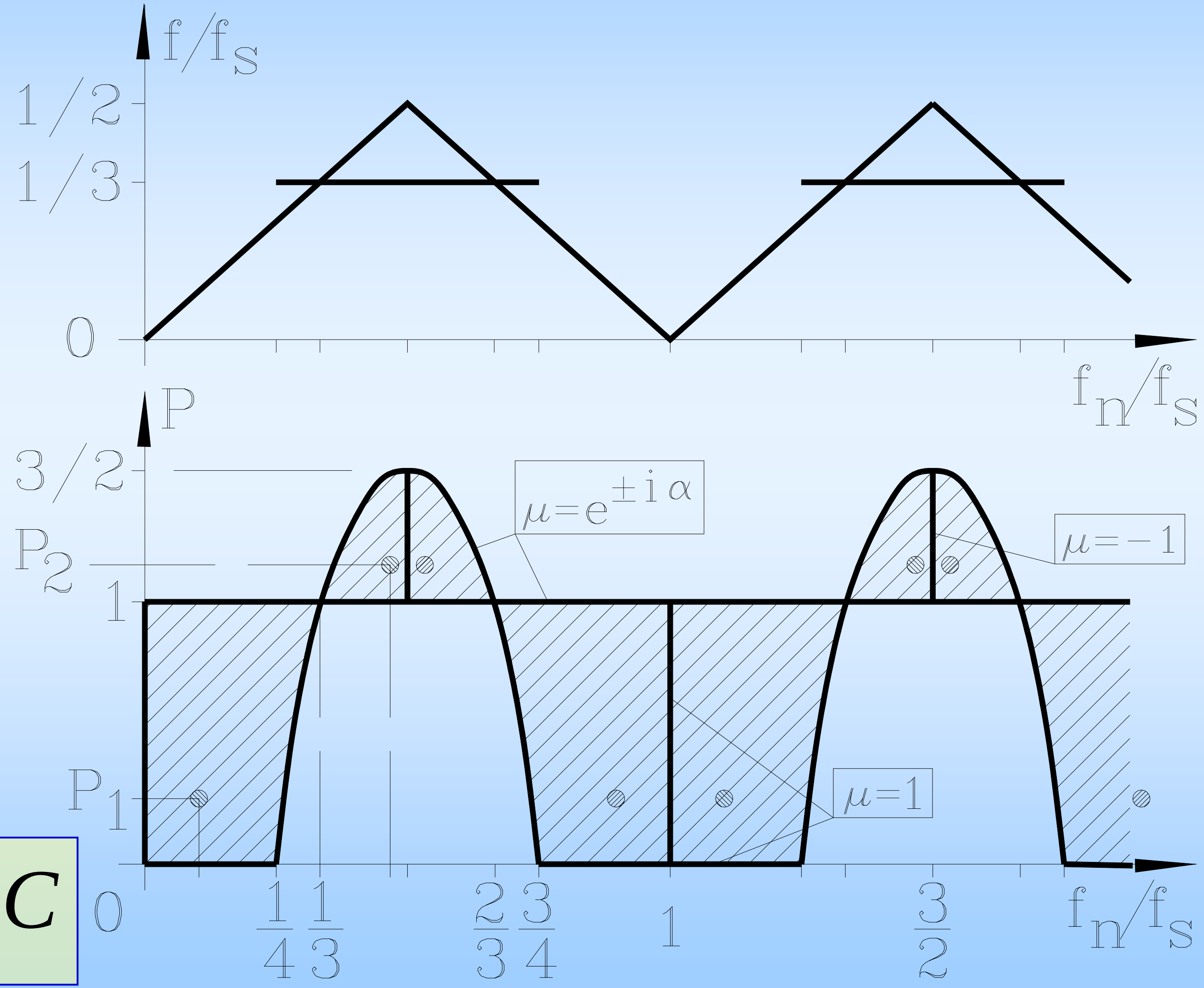
Vibration frequency: $0 < f < f_s/2$

Maximum gain:

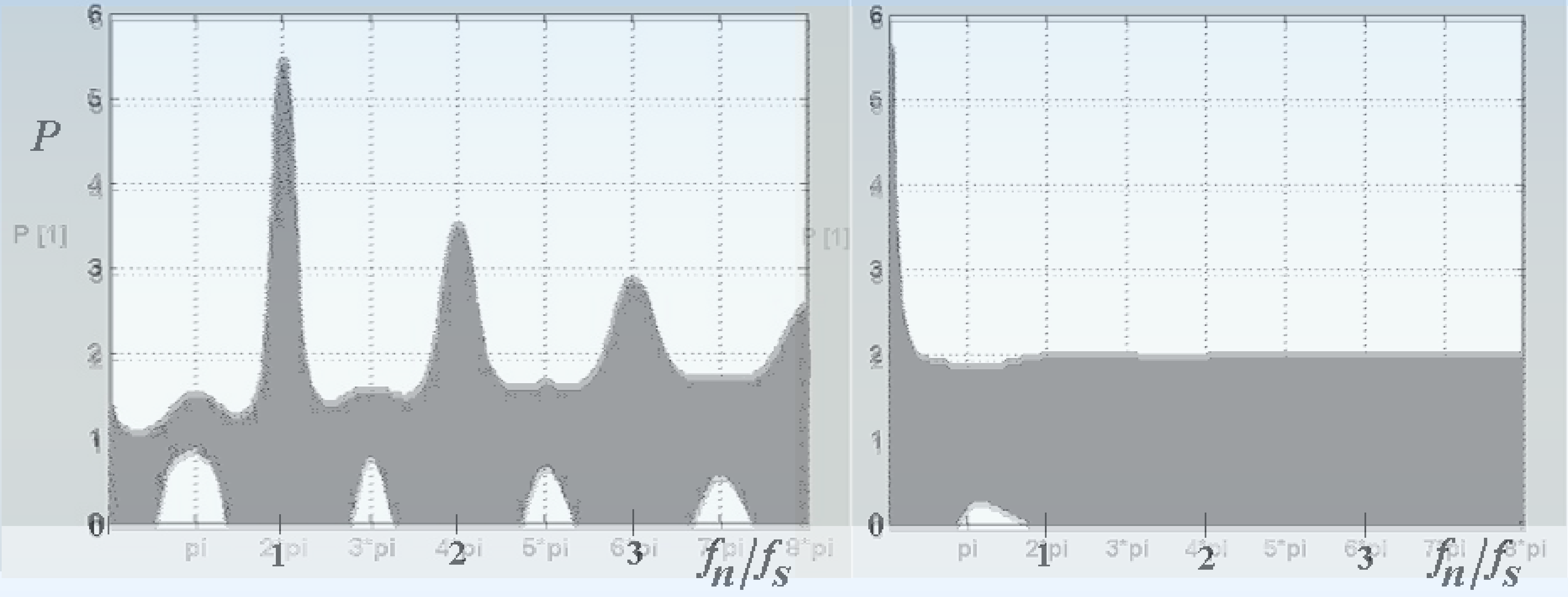
$$P_{\max} = 1.5$$

Minimum force error:

$$\Delta_{F,\min} \geq (2/3)C$$



Effect of viscous damping

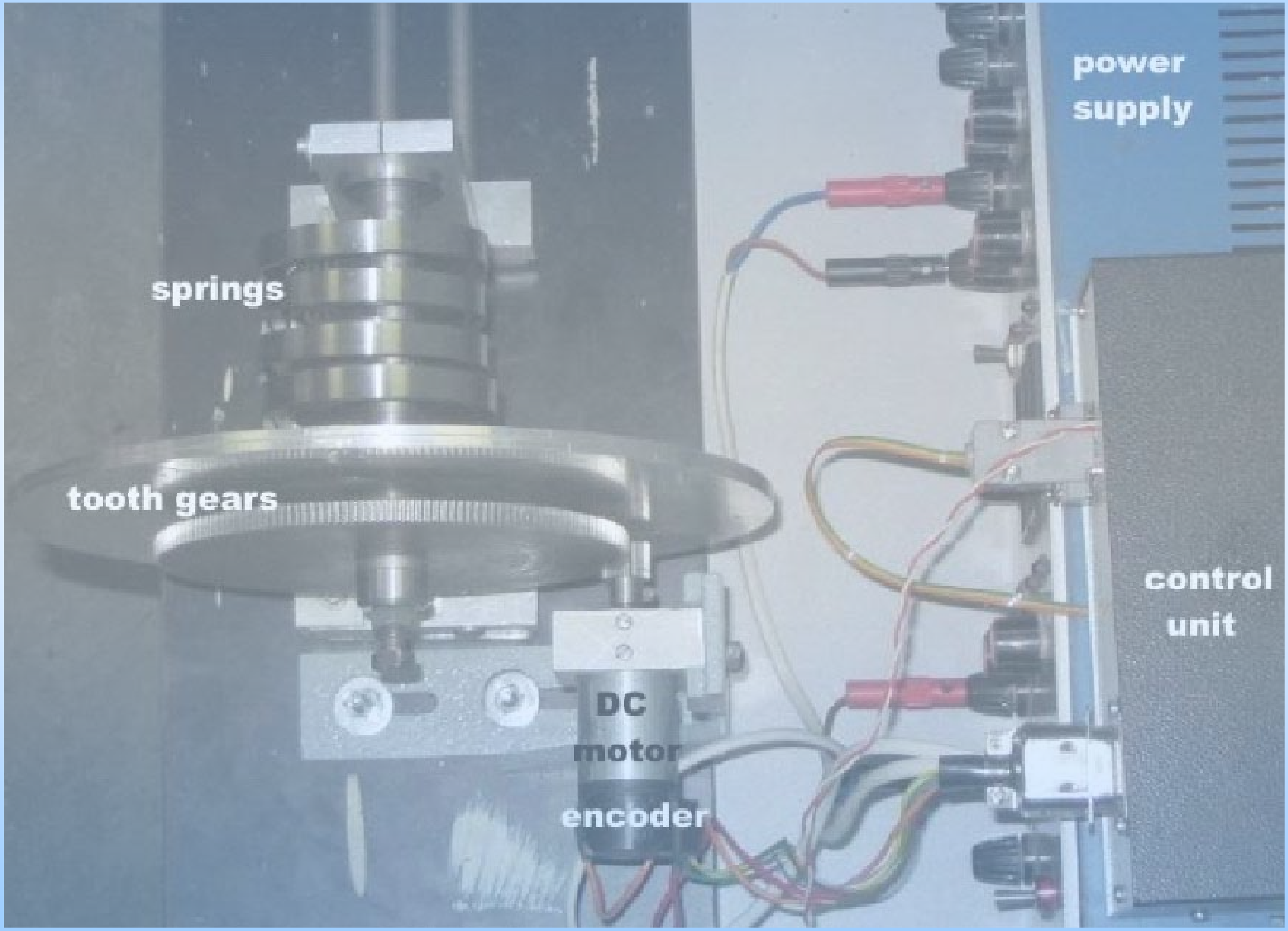


Damping ratio κ: **0.04** and **0.5**

$$x''(T) + 2\kappa(\omega_n\tau)x'(T) + (\omega_n\tau)^2 x(T) =$$

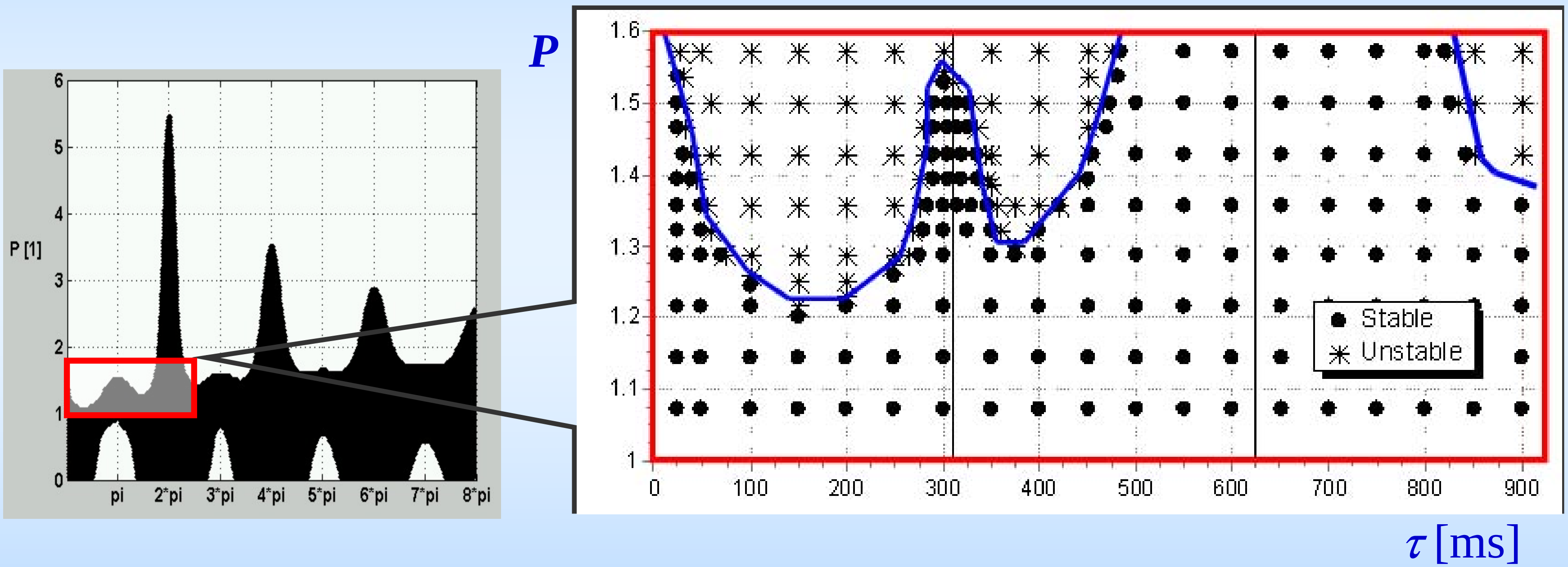
$$(\omega_n\tau)^2(1-P)x(j-1)$$

Experimental verification



Experimental verification

Damping ratio κ = 0.04



Differential gain D in force control

$$Q(t) \equiv -P\left(ky(t_j - \tau) - F_d\right) - D\left(k\dot{y}(t_j - \tau)\right) + ky(t_j - \tau)$$

$$x''(T) + (\omega_n \tau)^2 x(T) =$$

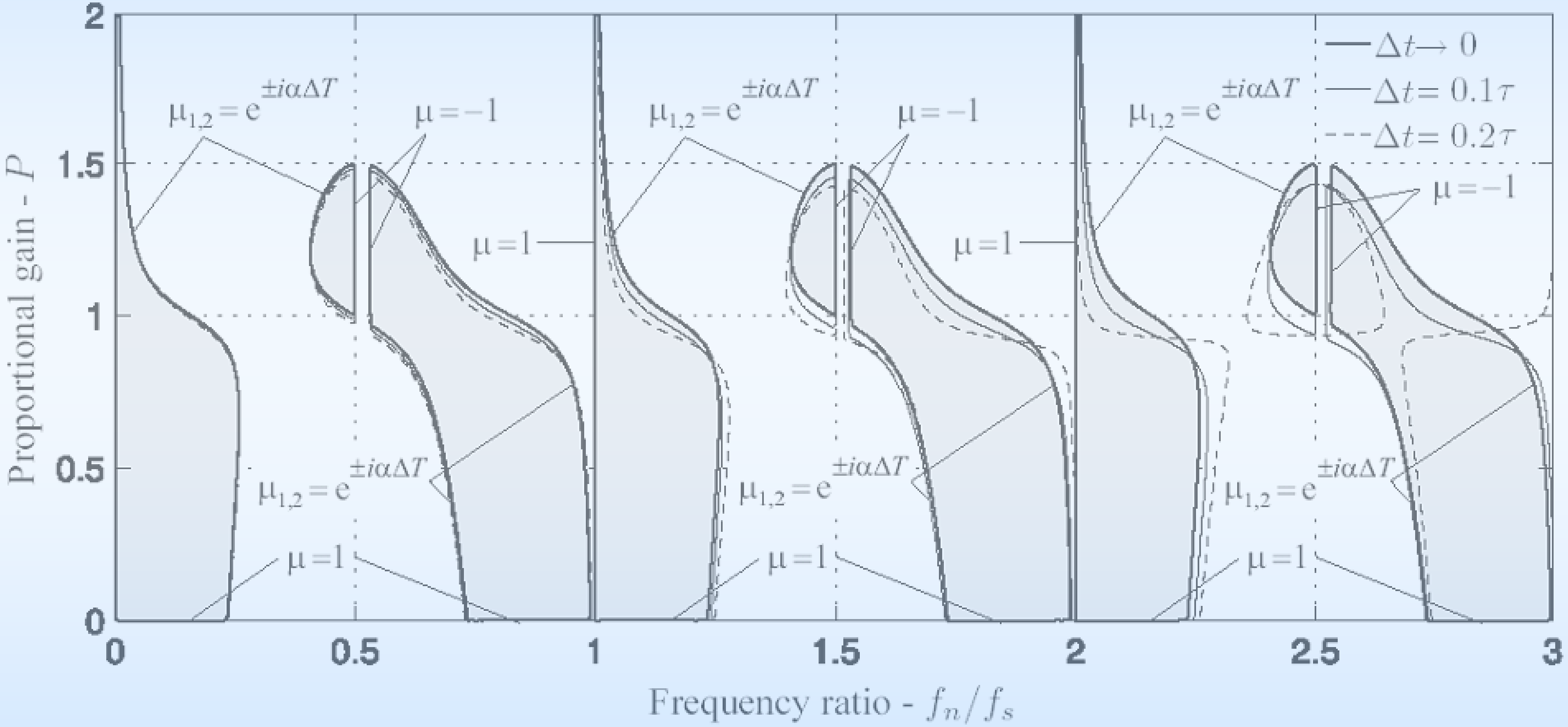
$$(\omega_n \tau)^2 (1 - P) x(j-1) - (\omega_n \tau) D x'(j-1)$$

Sampling at the force sensor with $\Delta t = q\tau$:

$$x'(j-1) \approx \frac{x(j-1) - x(j-1-q)}{q}$$

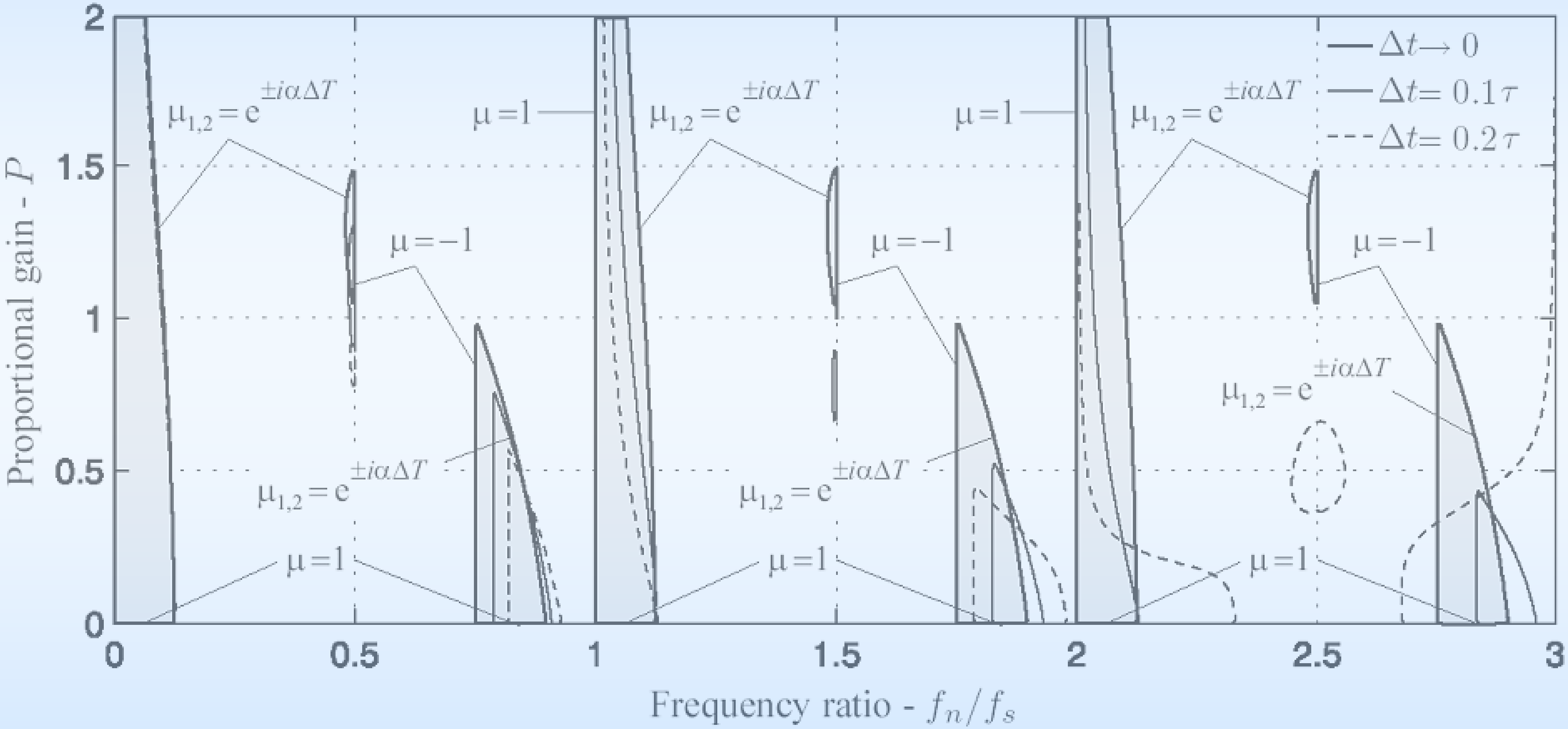
$$\mathbf{z}^j = \begin{pmatrix} x(j) & x'(j) & x(j-1) & x(j-1-q) \end{pmatrix}^T \Rightarrow \mathbf{z}^{j+1} = \mathbf{A}\mathbf{z}^j$$

Stability chart and bifurcations



Dimensionless differential gain $D\omega_n = 0.1$

Stability chart and bifurcations

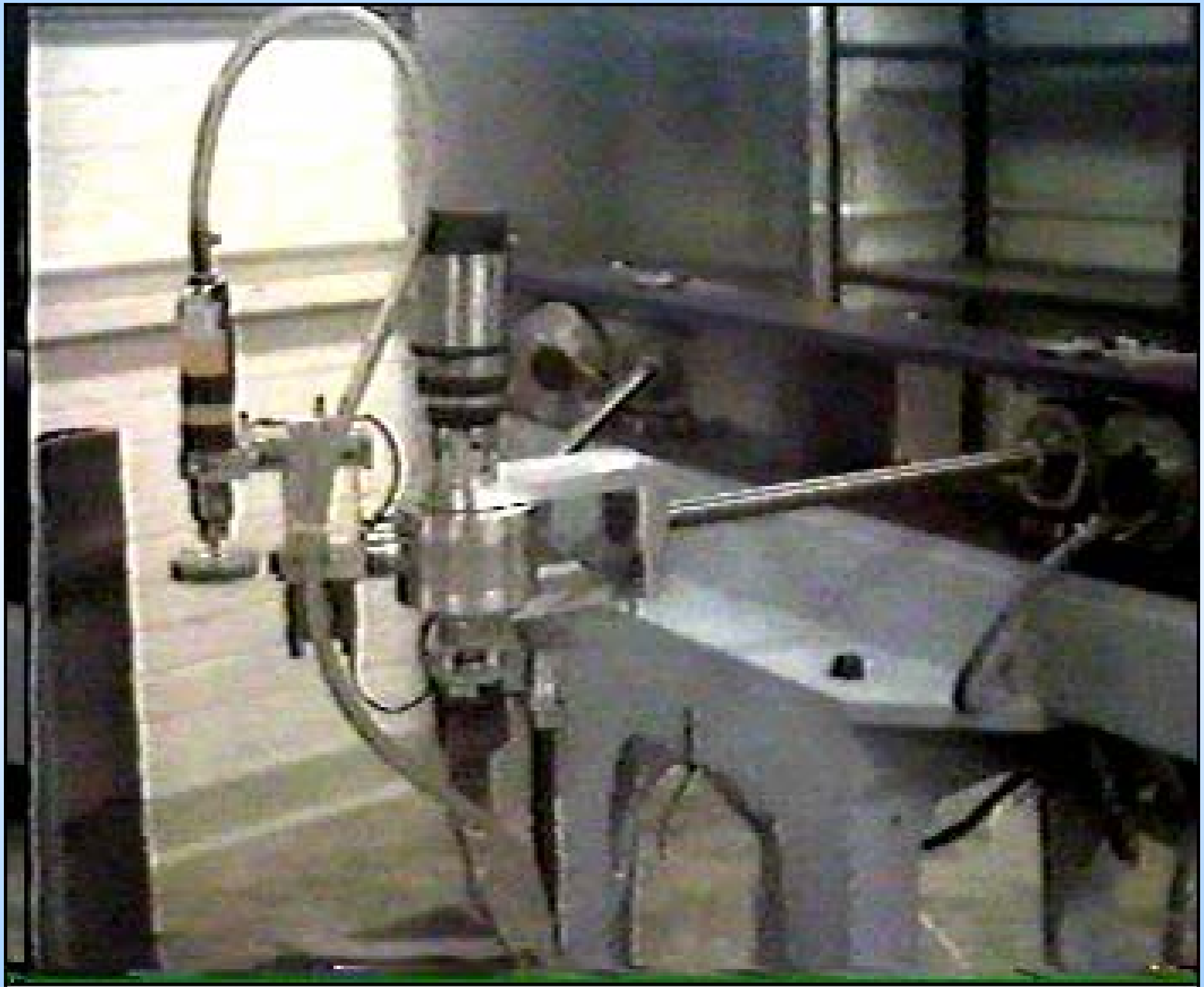


Dimensionless differential gain $D\omega_n = 1.0$

Conclusions on digital force control

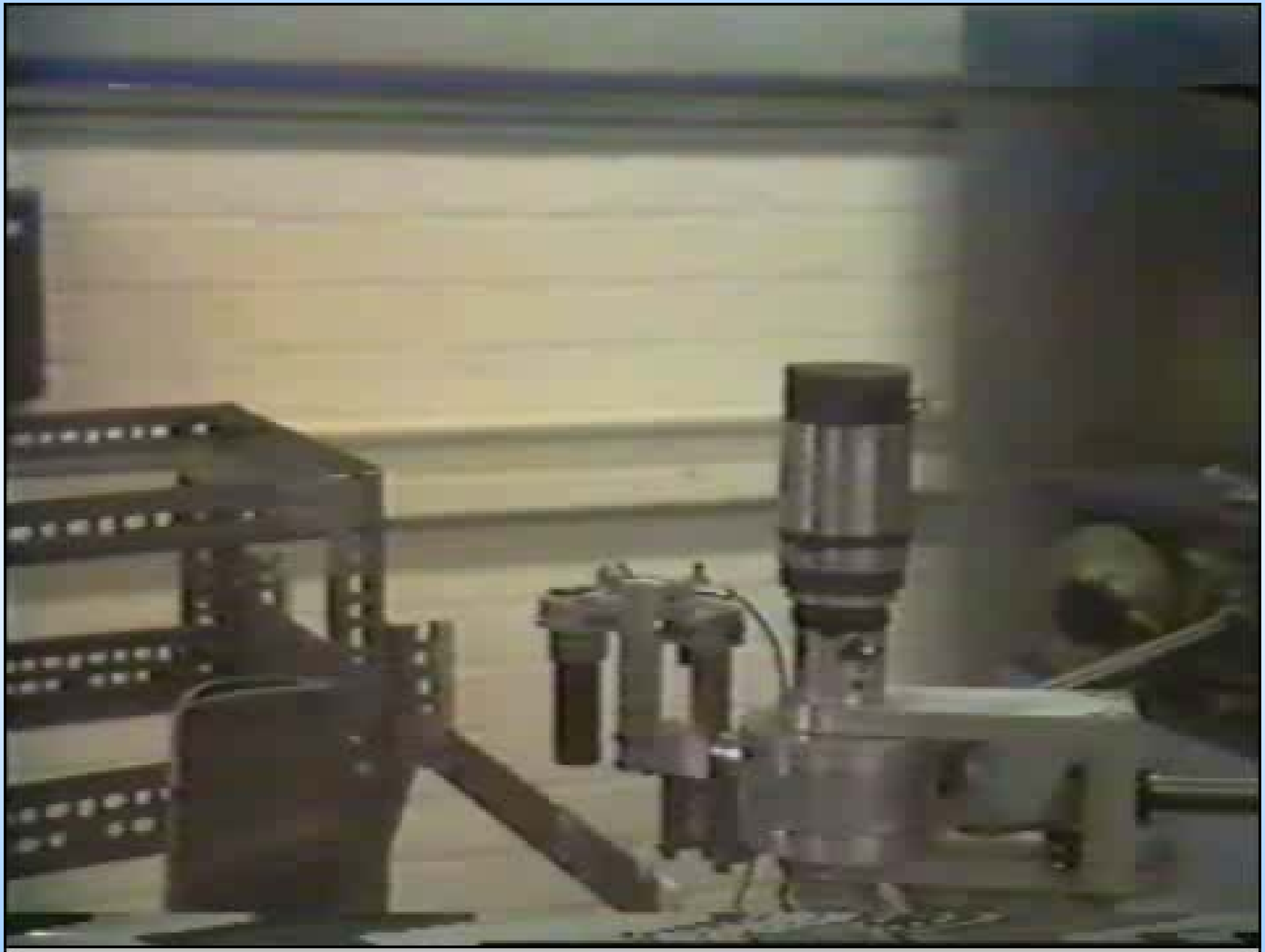
- All the 3 kinds of co-dimension 1 bifurcations arise in digital force control (Neimark-Sacker, flip, fold)
- Application of differential gain leads to loss of stable parameter regions
- Force derivative signal can be filtered with the help of sampling, but stability properties do not improve
- Do not use differential gain in force control

Turbine blade polishing



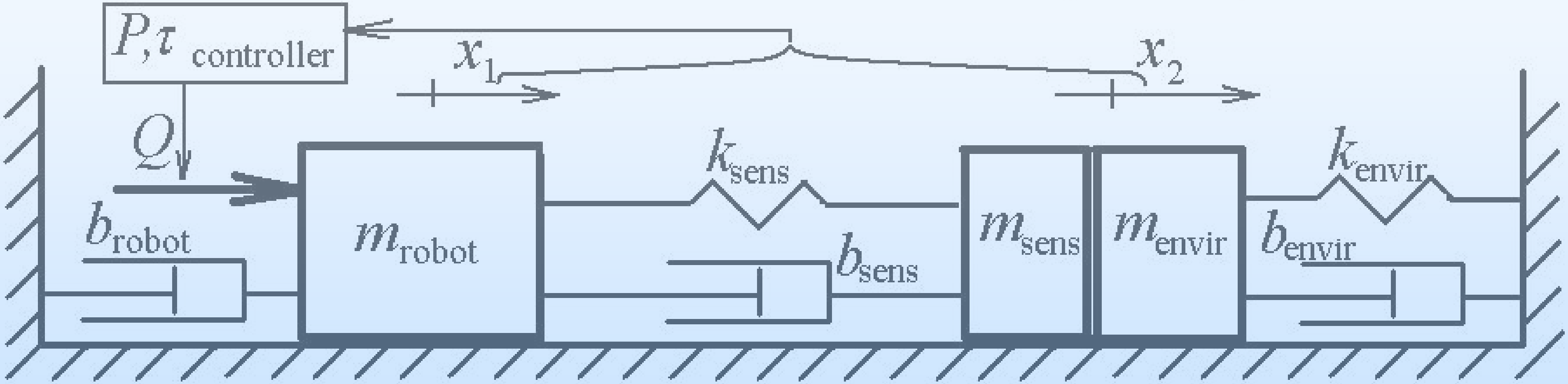
Eroszab.mpg

Stability problems along the blade

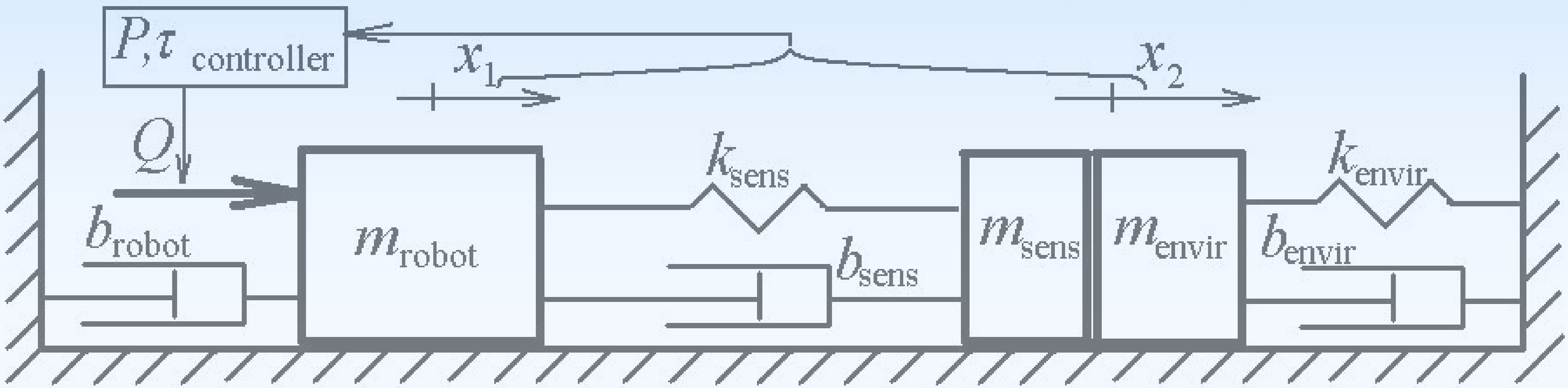


erorudon.avi

Turbine blade polishing



Mechanical model of polishing



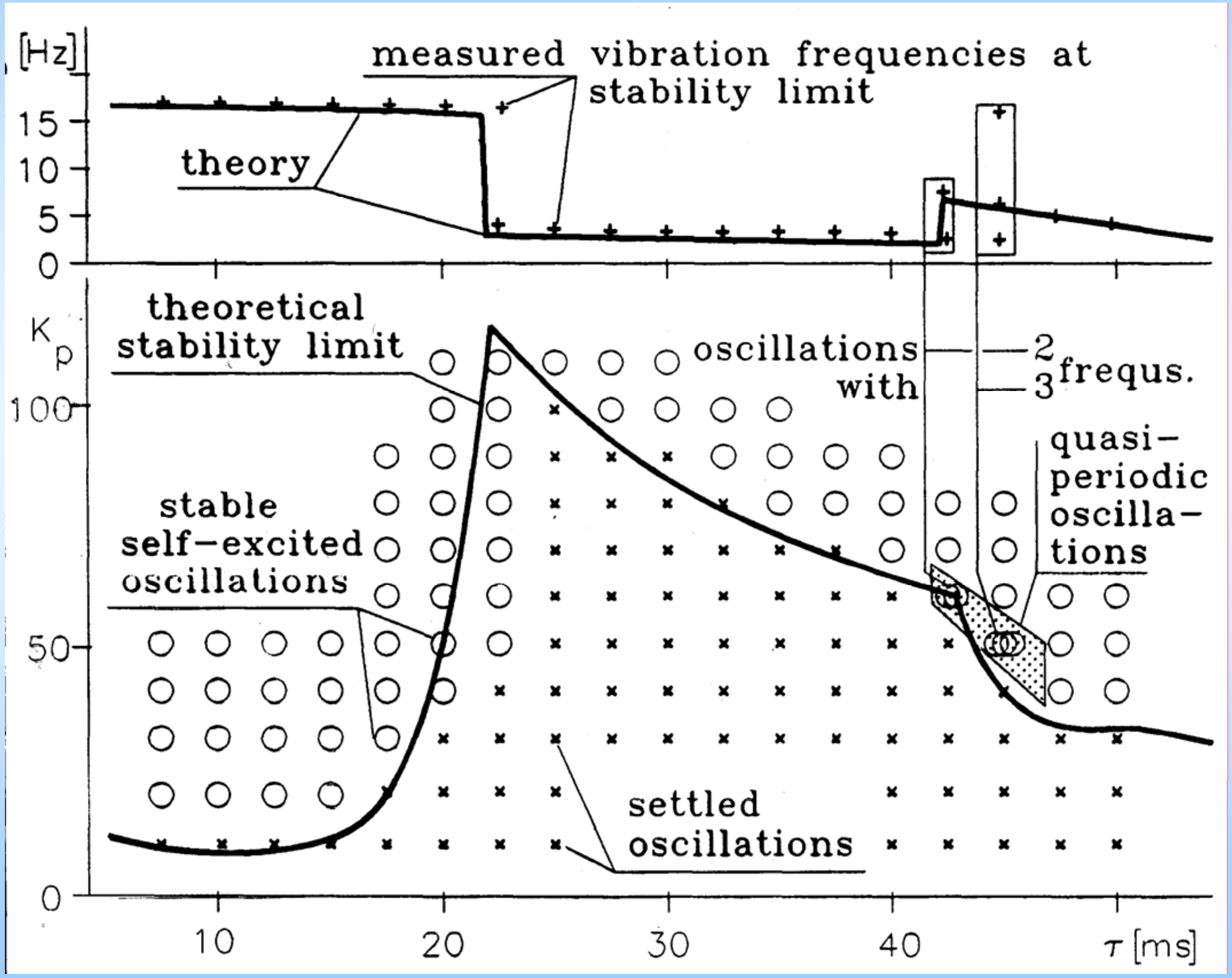
$m_r = 2500$ [kg] $b_r = 32$ [Ns/mm] $C = 150$ [N]

$m_s = 0.95$ [kg] $b_s = 2$ [Ns/m(!)] $k_s = 45$ [Ns/mm]

$m_e = 4.43$ [kg] $b_e = 3$ [Ns/m(!)] $k_s = 13$ [Ns/mm]

$F_d = 50$ [N]

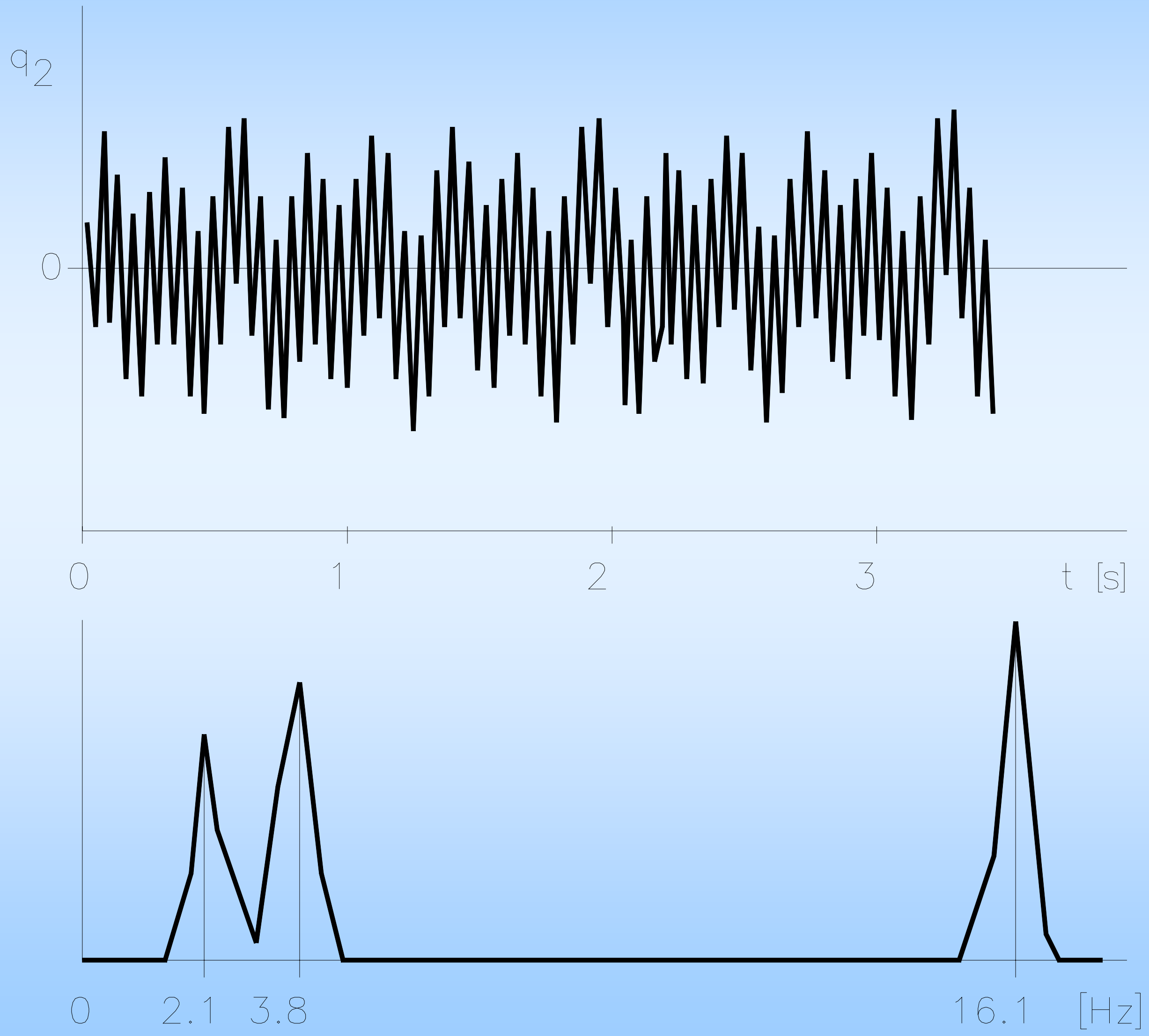
Experimental stability chart



The quasi-periodic oscillation

erokvazi.avi

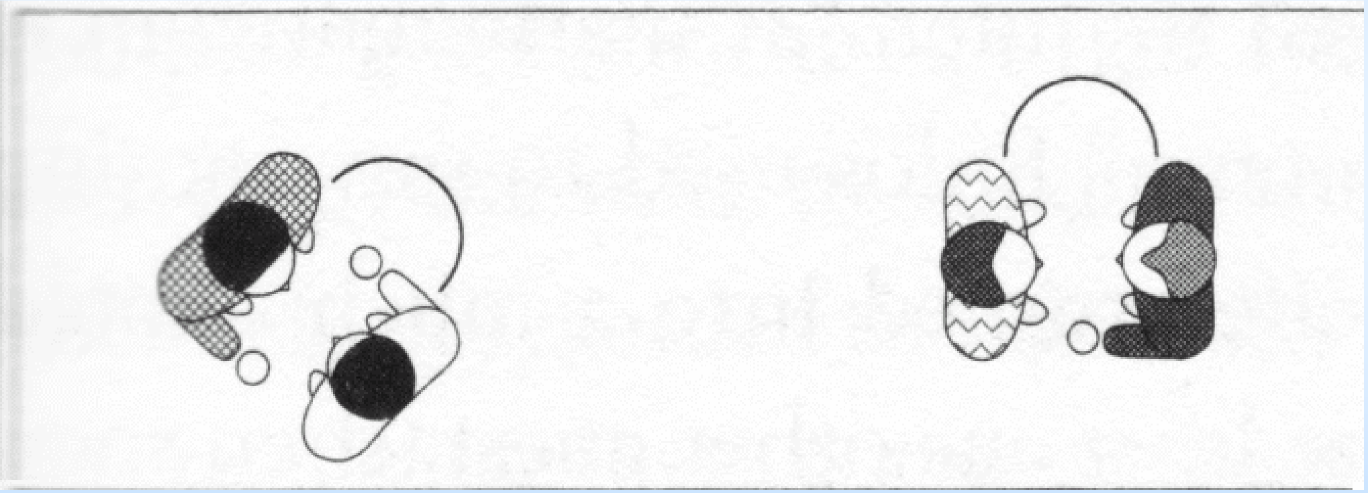
Time-history and spectrum



Human force control

"Good memory causes trouble"

Meeting others on narrow corridors $\Rightarrow$ oscillations



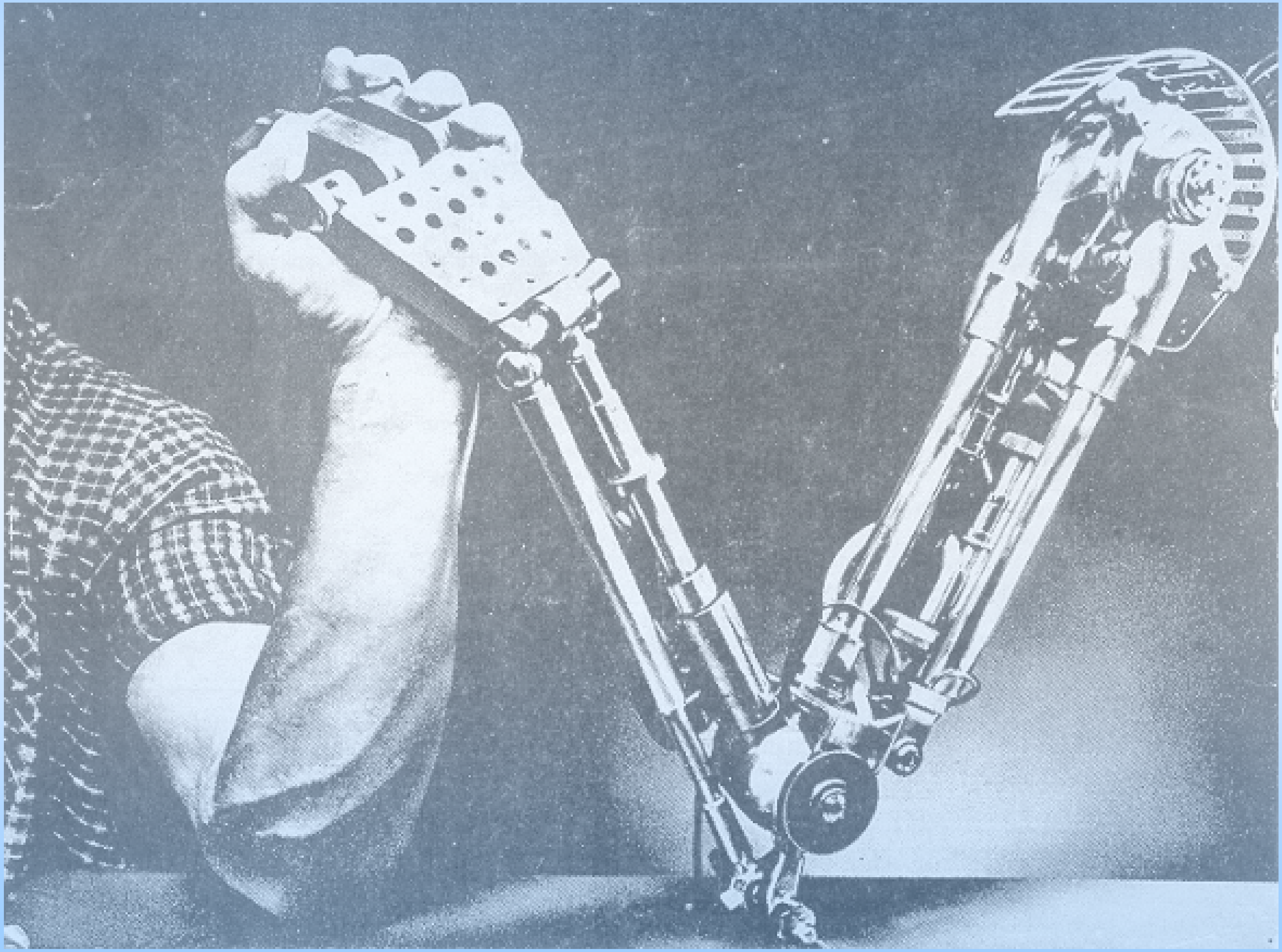
caused by the delay of our reflexes

Why do we **"shake"** hands?

Hemingway, E., The Old Man and the Sea (1952)

Santiago plays arm-wrestling in a pub of Casablanca

Human-robotic force control



Dexter cartoon

Rehabilitation robotics

- EU-V: **ReHaRob**
- (Budapest, Cardiff, Russe, Zebris)
- Upper limb motion therapy for hemiparetic patients
- Force control related vibrations

Rehabilitation robotics

IRB 1400 H

Frame

Stack lights

Touch screen

Control panel

Operation mode selector

Safety switch

Etc.

Couch

Instrumented orthosis

Start pedal

IRB 140

Similar projects

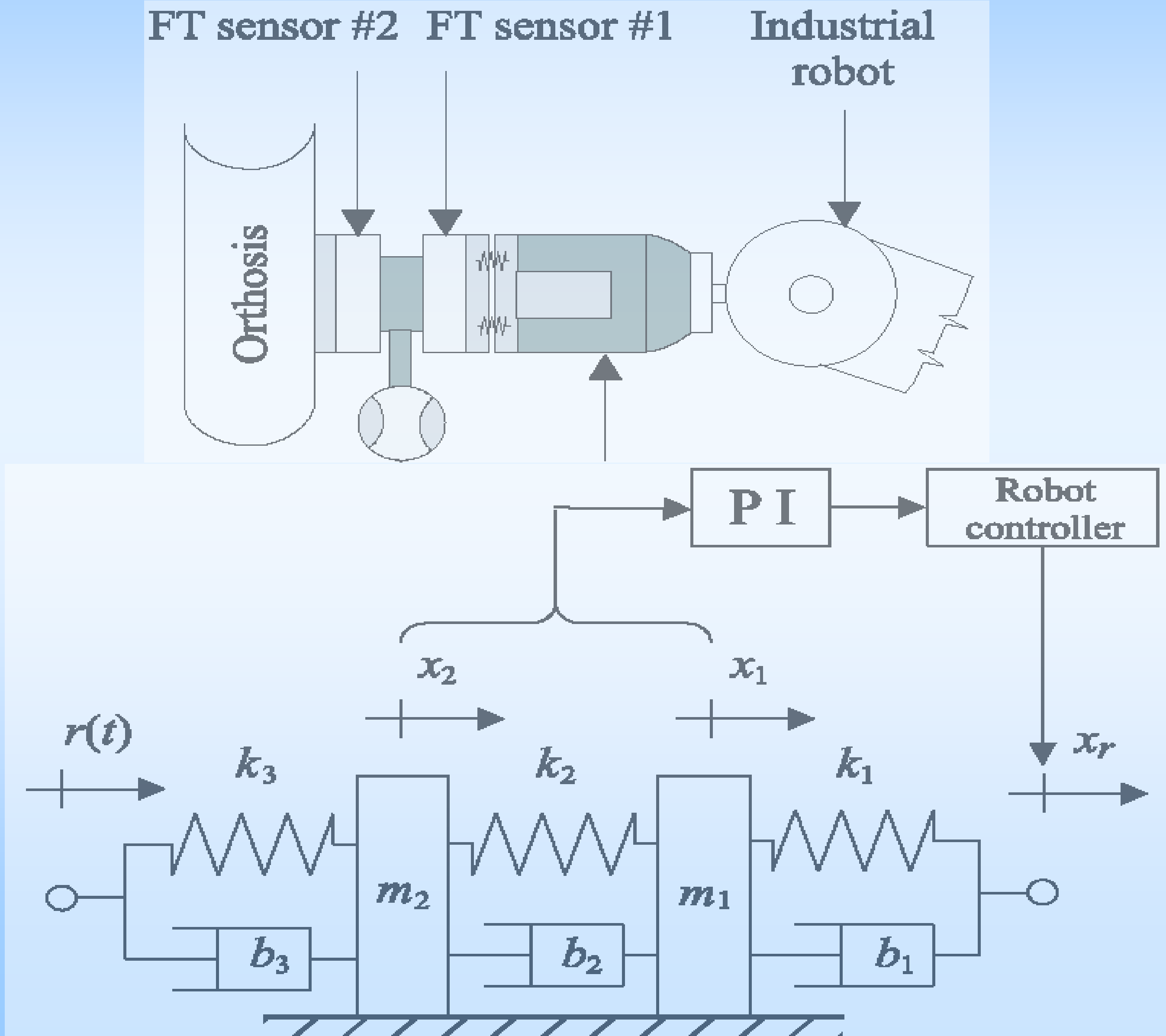
Newcastle (UK) MULOS   Stanford (USA) MIME

MIT (USA) MANUS   Reading (UK) Gentle/s

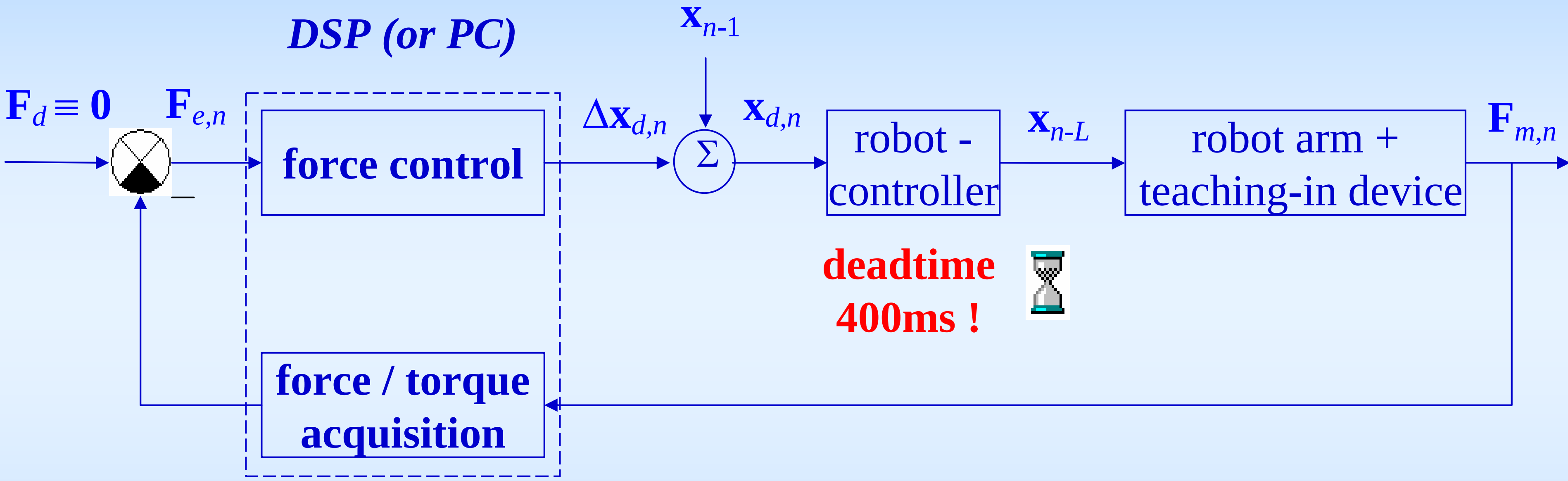
The safety relaxer

The instrumented orthosis

Mechanical model of relaxer



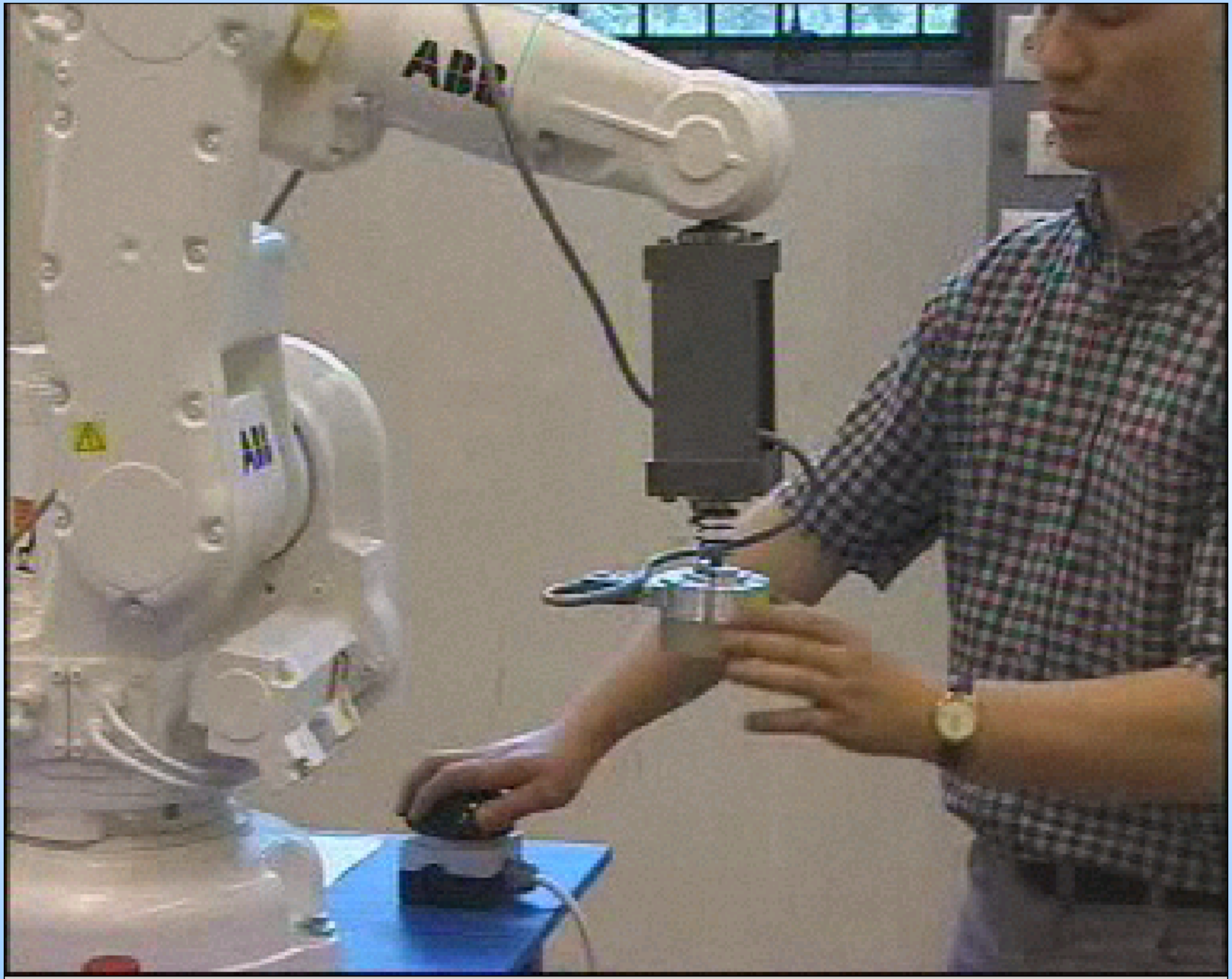
Force control during teaching-in



Sampling time at outer loop with $\tau \approx 60$ [ms]

Sampling time at force sensor with $\Delta t \approx 4$ [ms]

Delay and vibrations



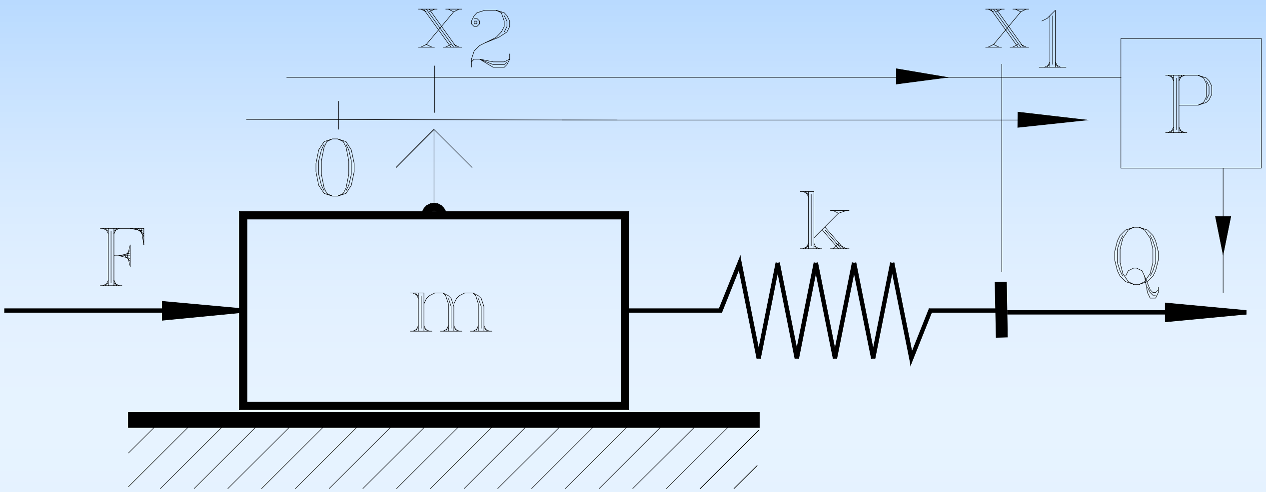
deadtime.avi

Delay and vibrations

Delay and vibrations

teachin.avi

1DoF modeling of digital control



$$m\ddot{x}_2(t) = k(x_1(t) - x_2(t))\,, \qquad t \in [t_j, t_j + \tau)$$

$$\dot{x}_1(t) = \underbrace{-Pk\Big(x_1(t_j - \tau) - x_2(t_j - \tau)\Big)}_{} + \dot{x}_2(t_j - \tau)$$

force error: $F_e(t_j - \tau)$

dimensionless gain: $p = Pk / \omega_n$

Stability chart and force error

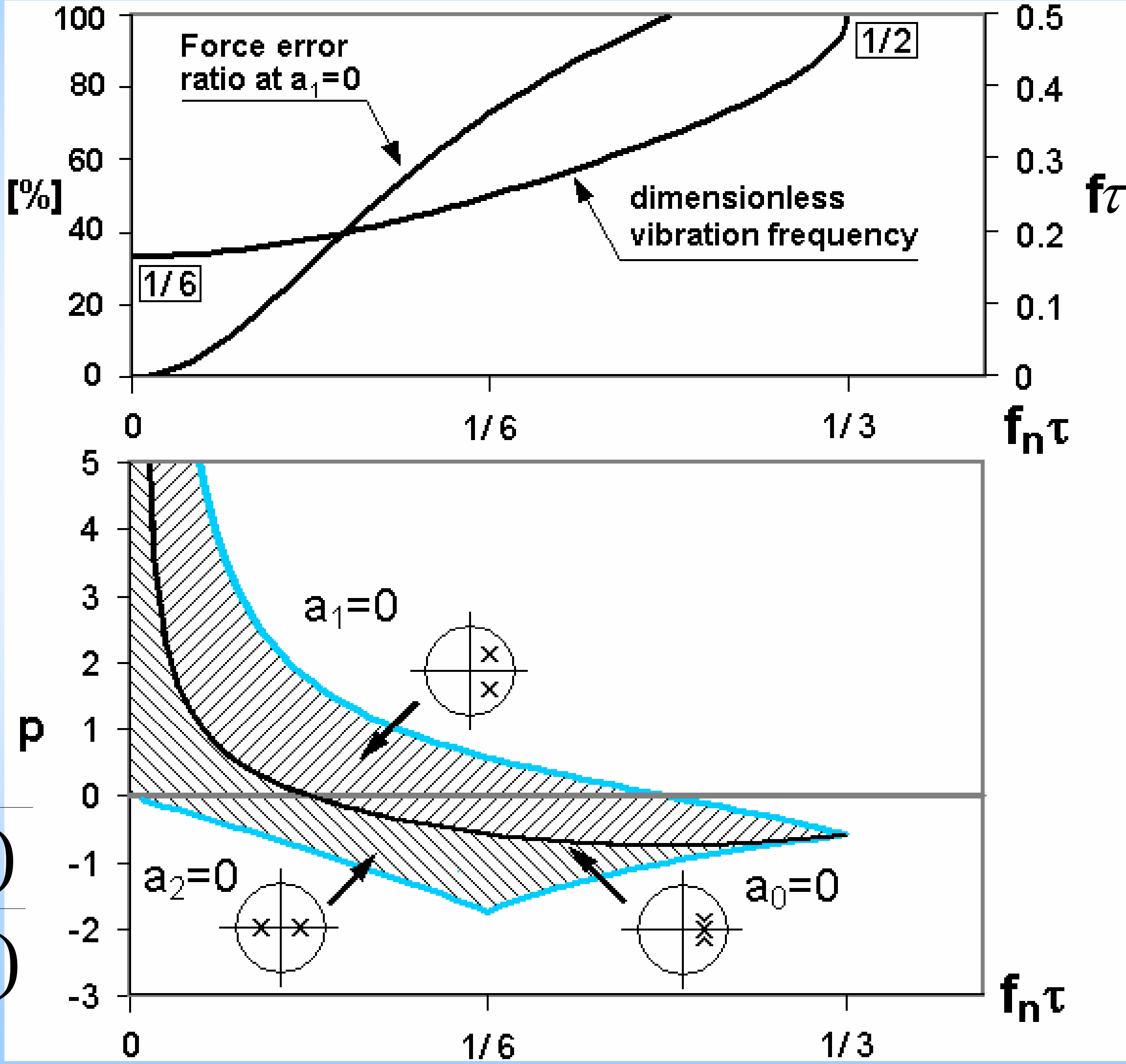
Force error ratio

$$\frac{3\omega_n\tau\tan(\omega_n\tau)}{2+3\omega_n\tau\tan(\omega_n\tau)}$$

$$\frac{\omega_n\tau}{2\pi}=f_n\tau=\frac{f_n}{f_s}$$

Vibration frequency

$$\frac{1}{2}-\frac{1}{\pi}\mathrm{atn}\sqrt{\frac{1+2\cos(\omega_n\tau)}{3-2\cos(\omega_n\tau)}}$$



Real parameter case study

Data:

$\tau = 30$ [ms]

$P = 0.012$ [s/kg]

$\Rightarrow \Delta_F / F_0 = 60$ [%]

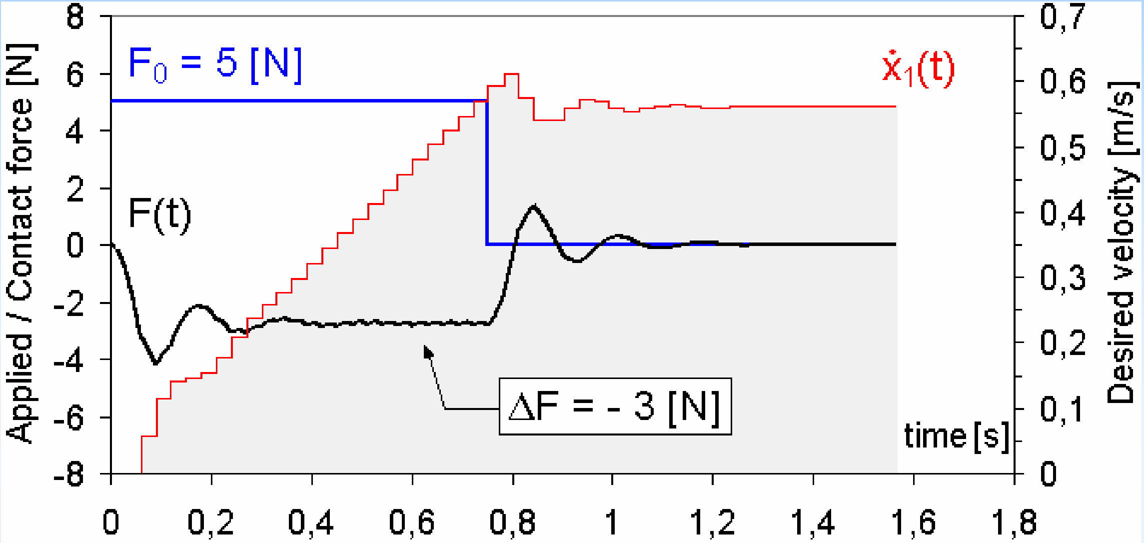
$m = 3$ [kg]

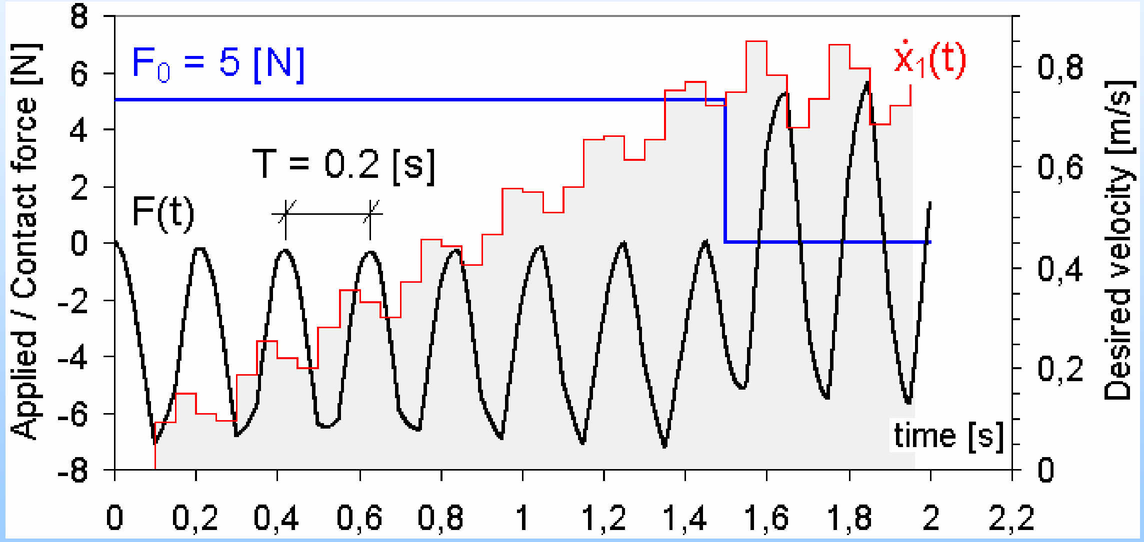
$k = 1200$ [N/m]

$\tau = 50$ [ms]

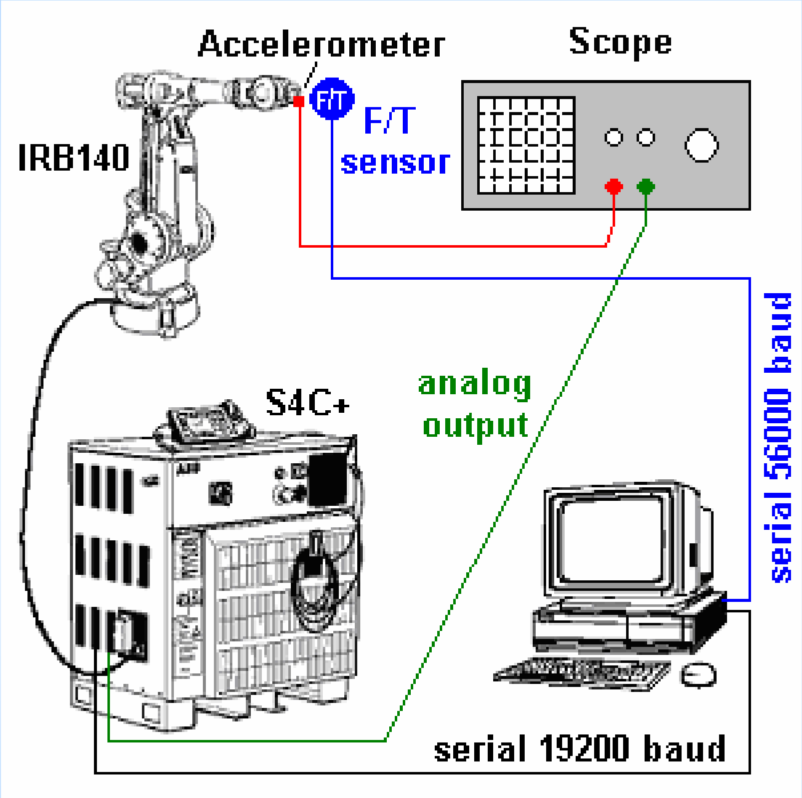
$P = 0.010$ [s/kg]

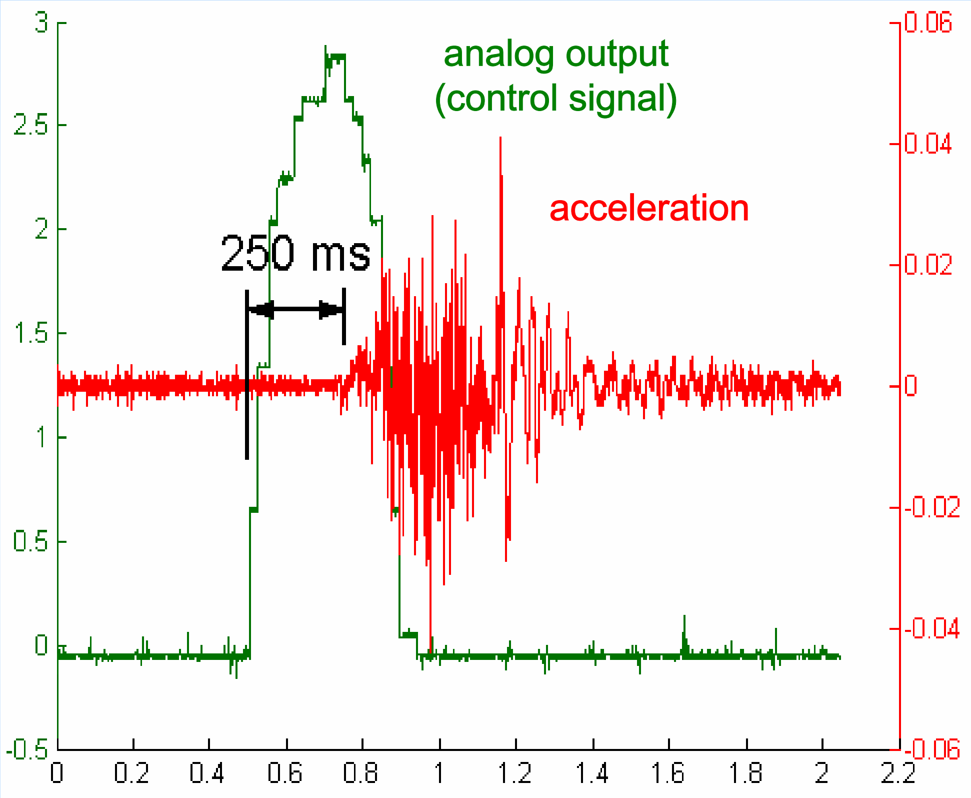
$\Rightarrow f = 5$ [Hz]





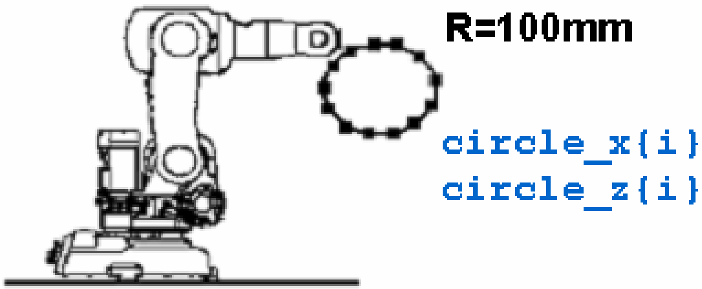
Experiments on additional dead time



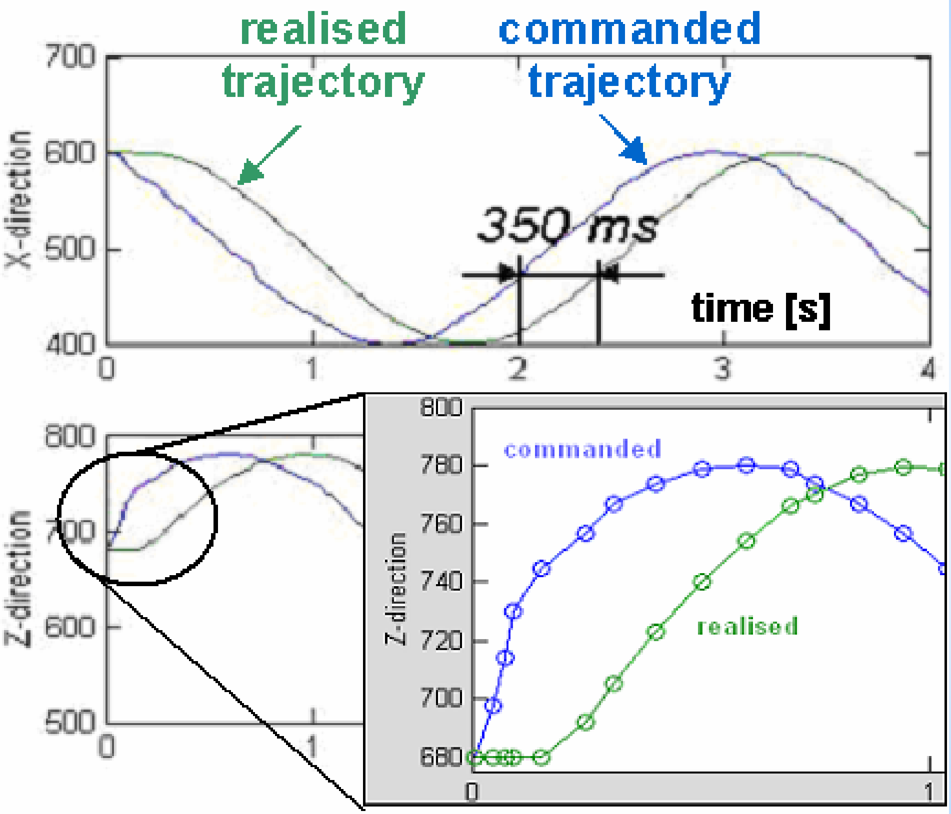


S4C+ controller on ABB robots have large delay due to its advanced path-planner

Experiments on additional dead time



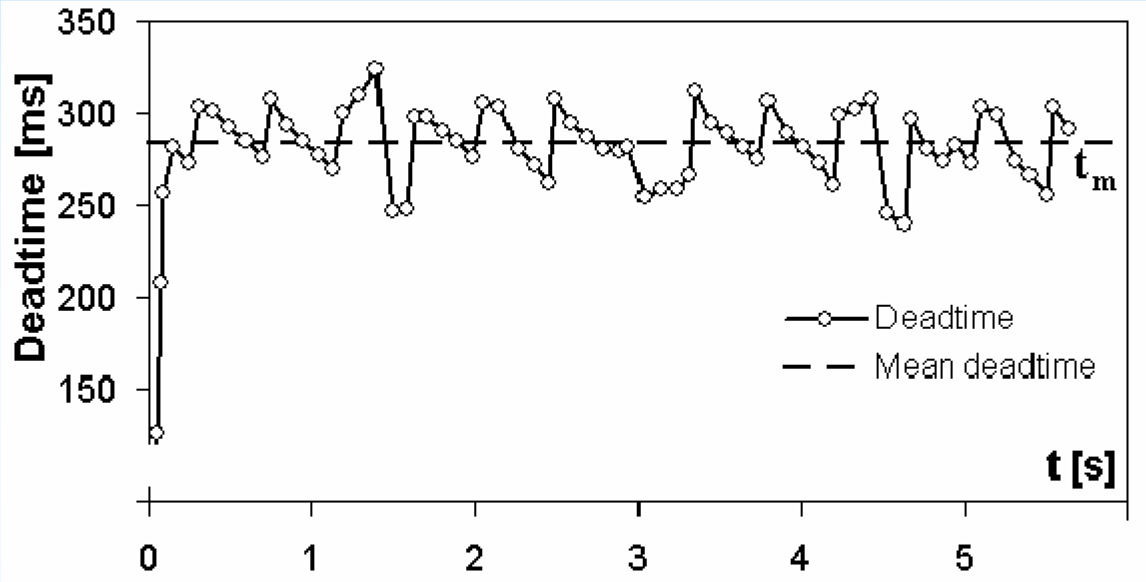

```
FOR i FROM 1 TO 72
  next_pos.trans.x:= circle_x(i);
  next_pos.trans.z:= circle_z(i);
  MoveL next_pos,v200,z5,tool0;
  cur_pos:=CRobT();
  real_pos(i):=cur_pos.trans;
  time(i):=ClkRead(timer);
ENDFOR
```



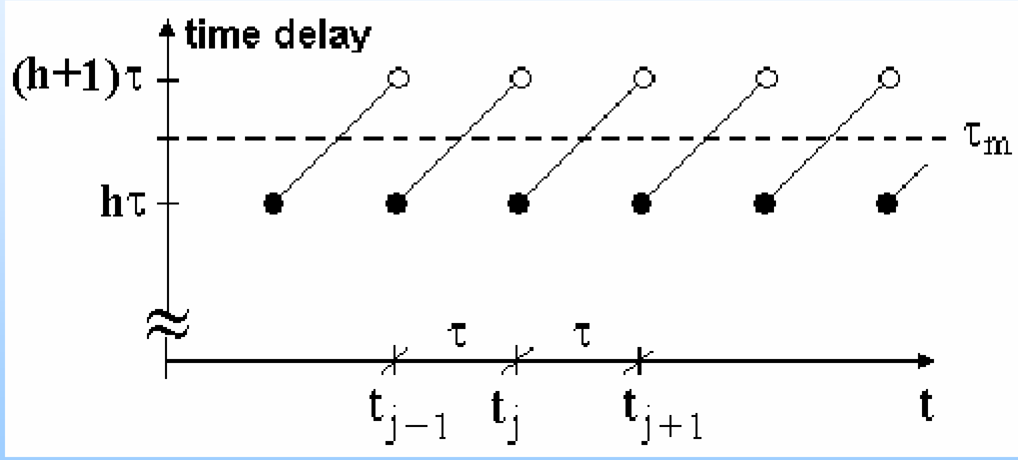
At this speed relative displacement needs ≈100[ms] so the additional dead time is about 250 [ms]

Modeling sampling and dead time

Random delay measurements



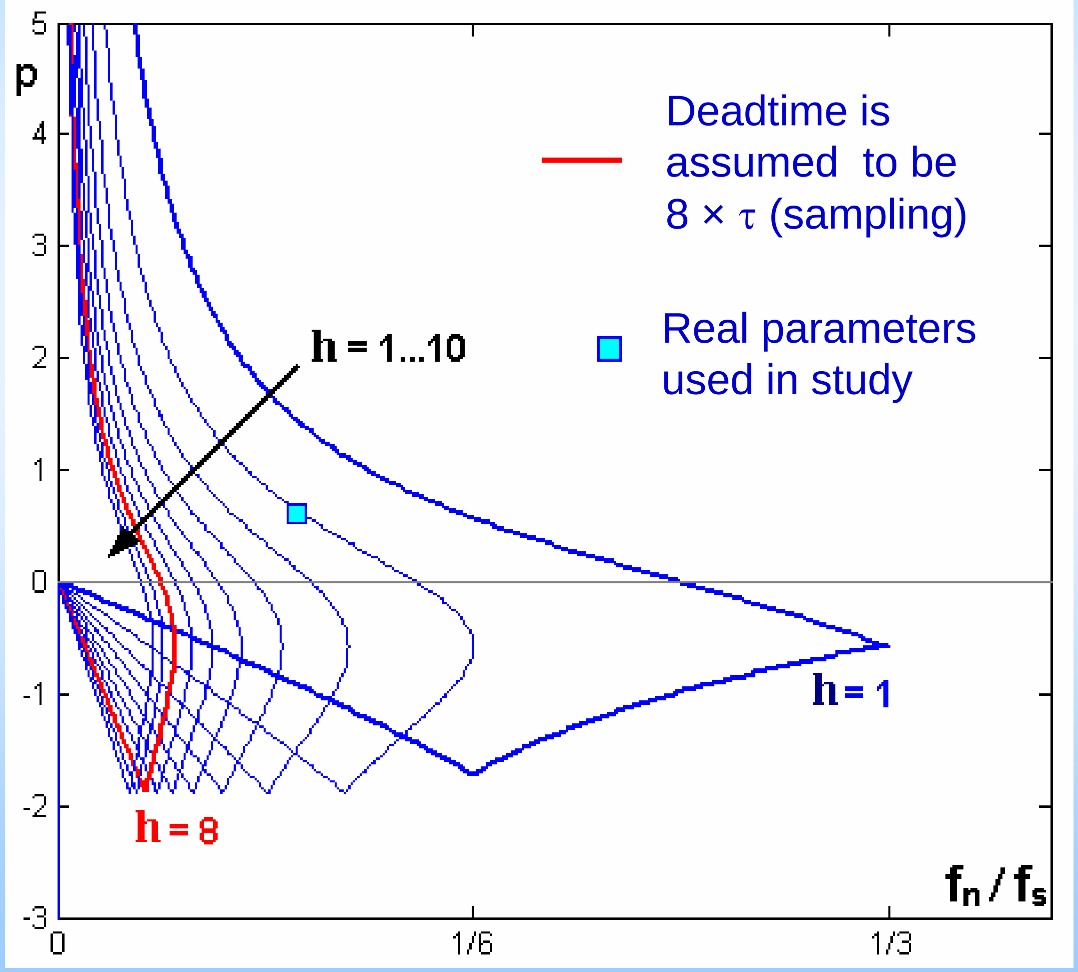
Combined delay and sampling model



Stability of rehabilitation robot

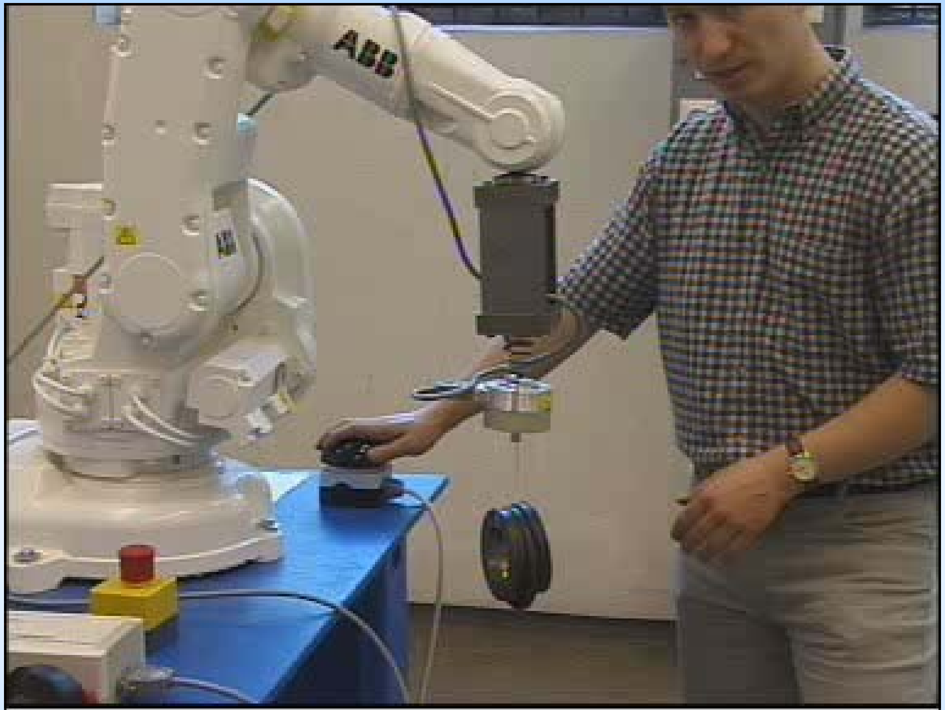
$$\dot{x}_1(t) = -Pk\left(x_1(t_j - h\tau) - x_2(t_j - h\tau)\right) + \dot{x}_2(t_j - h\tau)$$

$$t \in [t_j, t_j + \tau)$$



The unmodelled damping of the human palm and other viscous effects help to stabilize at chosen parameters

Experiments on stability



clip3.avi

clip6b.avi

stability problems with patient's mass

stable behavior in 6 DoF

plbackas.avi