How delay equations arise in Engineering?

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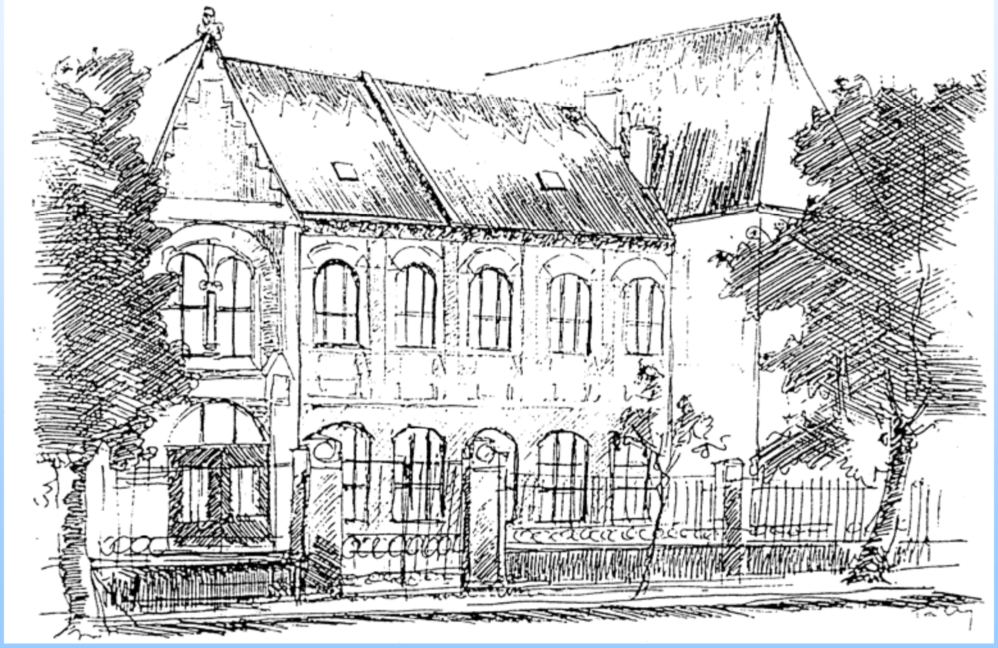


Contents

Answer: Delay equations arise in Engineering…

… by the information system (of control), and
by the contact of bodies.

- Linear stability & subcritical Hopf bifurcations
- Robotic position and force control
- Balancing – human and robotic
- **Contact problems**
- **Shimmying wheels (of trucks and motorcycles)**
- Machine tool vibrations

Stability of linear RFDEs of n DoF systems

Delayed mechanical systems include 2nd derivatives:

$$M\ddot{x}(t) + \int_{-h}^{0} \mathrm{d}B(\vartheta)\dot{x}(t+\vartheta) + \int_{-h}^{0} \mathrm{d}K(\vartheta)x(t+\vartheta) = 0$$

Trial solution: $\boxed{x(t) = A\mathrm{e}^{\lambda t}}$ $\quad A \in R^n$

Characteristic roots: Re $\lambda_j < 0$, j=1,2,… ⇔ stability

$$D(\lambda) = \det(M\lambda^2 + \int_{-h}^{0} \lambda \mathrm{e}^{\lambda\vartheta} \mathrm{d}B(t,\vartheta) + \int_{-h}^{0} \mathrm{e}^{\lambda\vartheta} \mathrm{d}K(\vartheta))$$

D-curves: $R(\omega) = \operatorname{Re} D(i\omega),\ S(\omega) = \operatorname{Im} D(i\omega),\ \omega \in [0,\infty)$

$$\left.\begin{aligned} &R(\rho_k) = 0, k = 1,\ldots,r: \quad S(\rho_k) \neq 0, k = 1,\ldots,r \\ &\sum_{k=1}^{r} (-1)^k \operatorname{sgn} S(\rho_k) = (-1)^n n \end{aligned}\right\}$$ ⇔ stability

Example with 1 DoF, $n = 1$

$$\ddot{x}(t) + c_0 x(t) = c_1 \int_{-1}^{0} w(\vartheta) x(t+\vartheta)\,\mathrm{d}\vartheta, \quad w(\vartheta) \equiv 1$$

$$D(\lambda) = \lambda^2 + c_0 - c_1 \int_{-1}^{0} e^{\lambda\vartheta}\,\mathrm{d}\vartheta = \lambda^2 + c_0 - c_1 \frac{1 - e^{-\lambda}}{\lambda}$$

$$R(\omega) = -\omega^2 + c_0 - c_1 \frac{\sin\omega}{\omega} \quad \Rightarrow \lim_{\omega\to+\infty} R(\omega) = -\infty$$

$$S(\omega) = c_1 \frac{1-\cos\omega}{\omega} \quad \Rightarrow S(\omega) > 0 \text{ for } \boxed{c_1 > 0},$$

$$\omega \neq 2k\pi, k = 0,1,\ldots$$

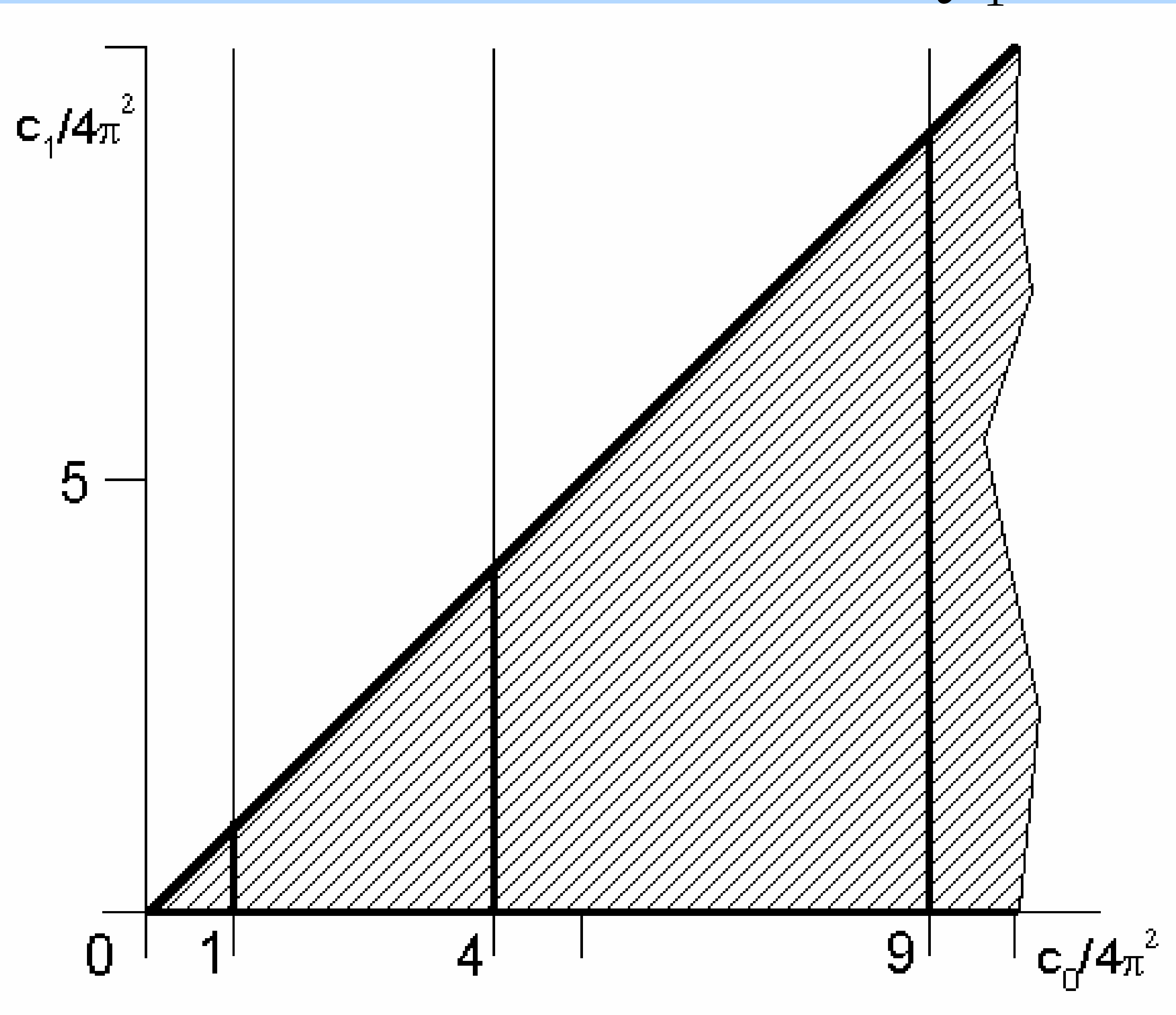
$$S(\rho_k) \neq 0, k = 1,\ldots,r \quad \Rightarrow R(2k\pi) = \boxed{-4k^2\pi^2 + c_0 \neq 0}$$

$$\sum_{k=1}^{r} (-1)^k \underbrace{\operatorname{sgn} S(\rho_k)}_{+1} = \underbrace{(-1)^n n}_{-1} \quad \Rightarrow r \text{ odd}$$

$$\Rightarrow \mathrm{R}(0) = \boxed{\mathrm{c}_0 - c_1 > 0}$$

Stability chart

$$\ddot{x}(t) + c_0 x(t) = c_1 \int_{-1}^{0} x(t+\vartheta)\,\mathrm{d}\vartheta$$



Shimmy

Quasi-periodicity!

Mechanical degrees of freedom?

Shimmy

video1

video2

Shimmy

Shimmy

TL1000 recalled

SUZUKI has finally recalled all TL1000s to fit steering dampers after police started probing two rider deaths, and widespread criticism of the bike's handling.

All TL owners in the UK have been contacted and urged to return their bikes to dealers to have the new damper system fitted free of charge.

Fears for the TL's stability hit the press after Simon Carolan-Evans, 36, was killed in North Yorkshire in April.

And last month Farshad Sanjoori, 29, was killed in Cobham, Surrey, after an accident on his TL1000.

Bike was first with the full story of the TL1000's potential handling problems after dozens of readers rang us with details of scary wobbles.

We took a TL1000 to independent suspension expert Dave Parkinson: his verdict was that the radical rear rotary damper is not compliant enough, the swingarm is too short, and the front suspension is poorly damped. He didn't suggest fitting a steering damper.

The next delivery of 500 TLs arrives from Japan this month. They will be fitted with the damper kits before going out to dealers.

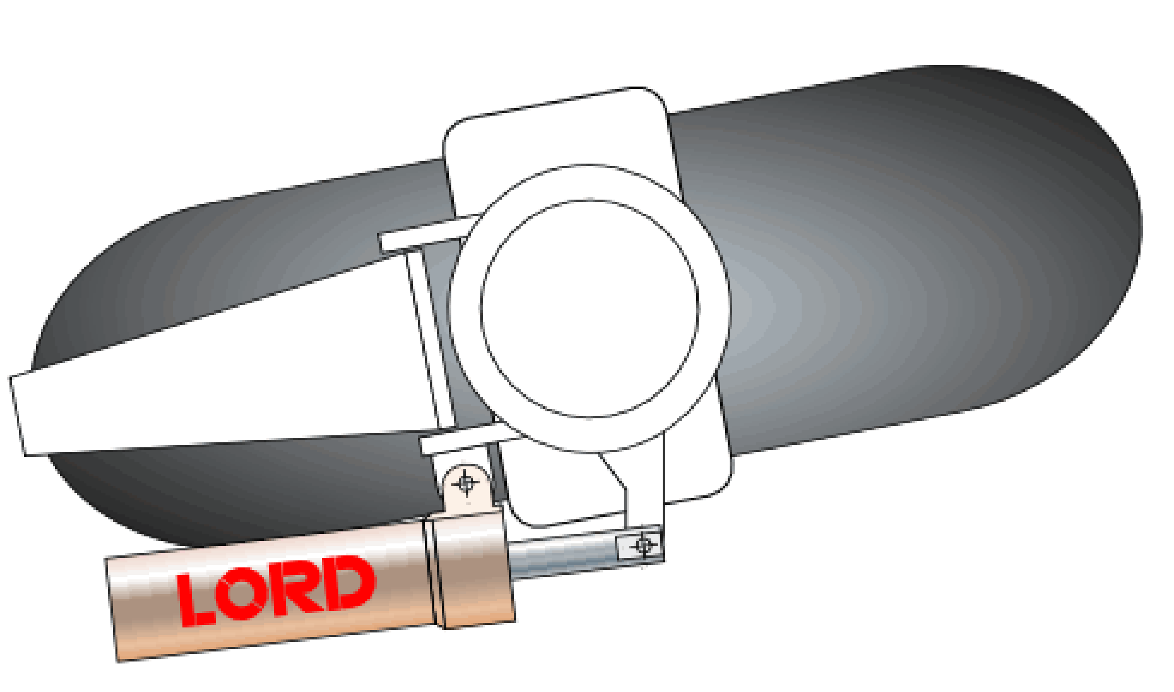
A Suzuki spokesman told Bike: 'The steering damper was a planned upgrade for the bike in 98. It's a reaction to comments made in letters to us and reports in the press.'

Shimmy

Shimmy

Shimmy

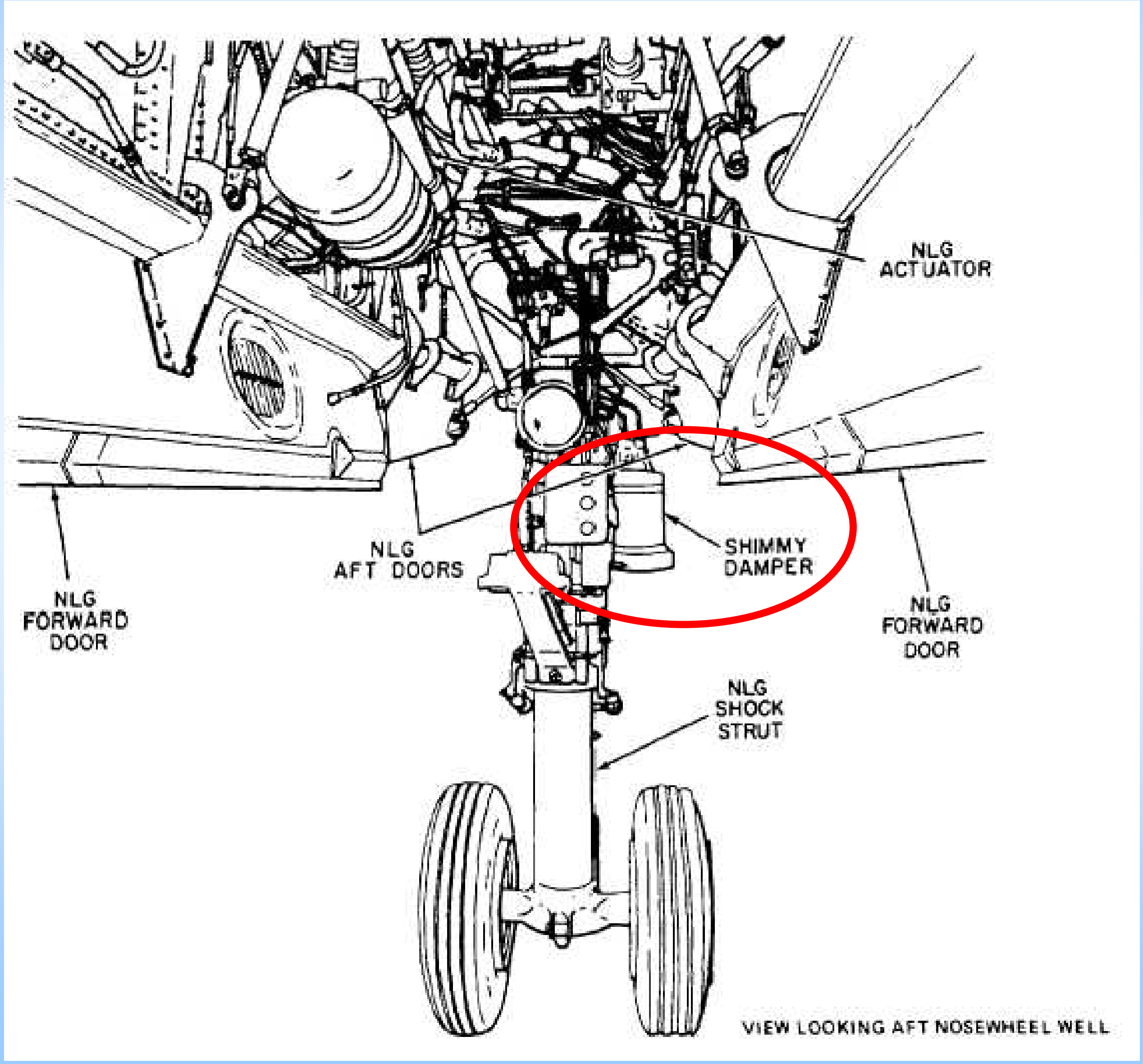
Shimmy



Shimmy on airplanes

Shimmy on airplanes

Shimmy on airplanes

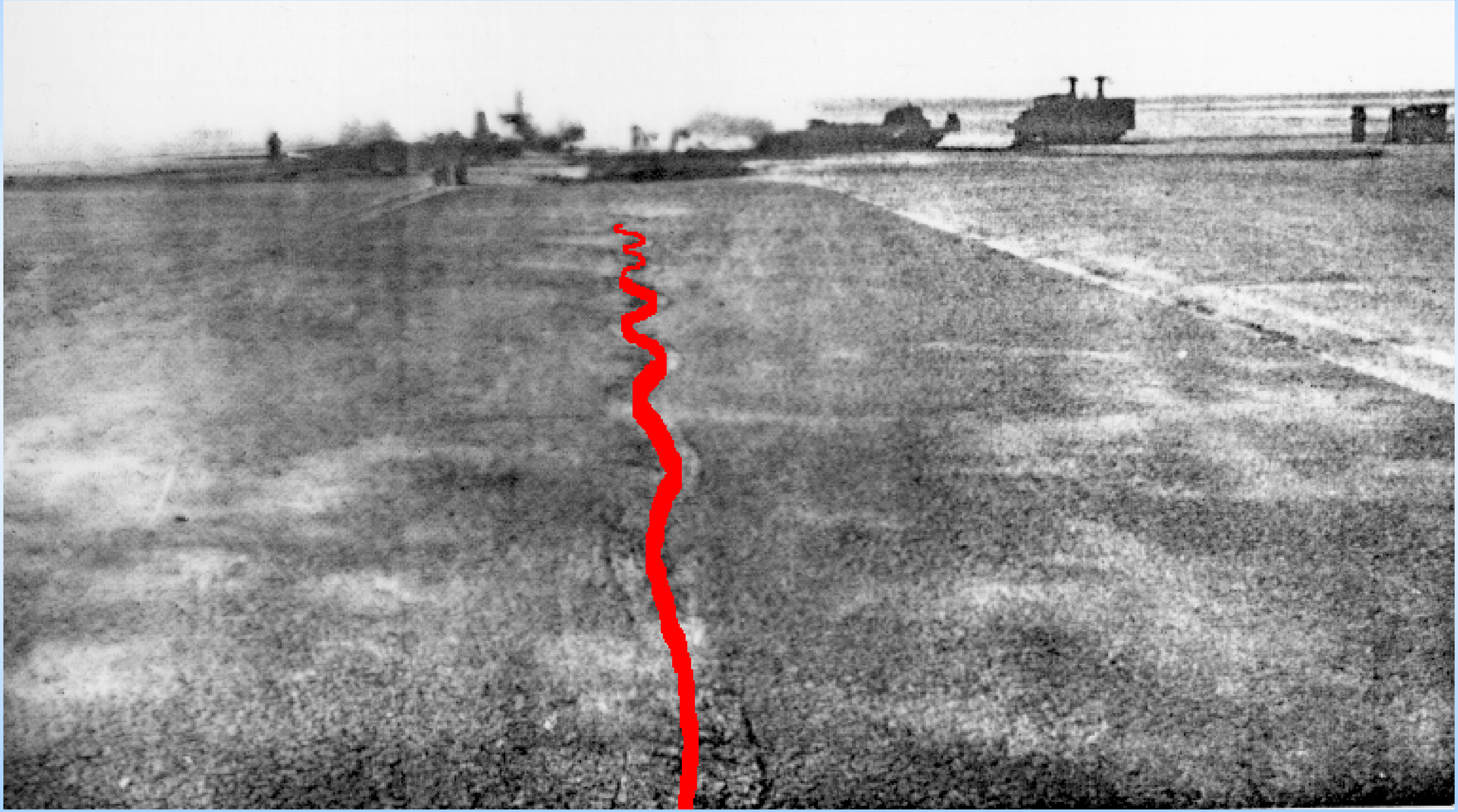


Shimmy on airplanes

Hawker 125

Shimmy on airplanes

Shimmy on airplanes



Concorde 2001

Concorde

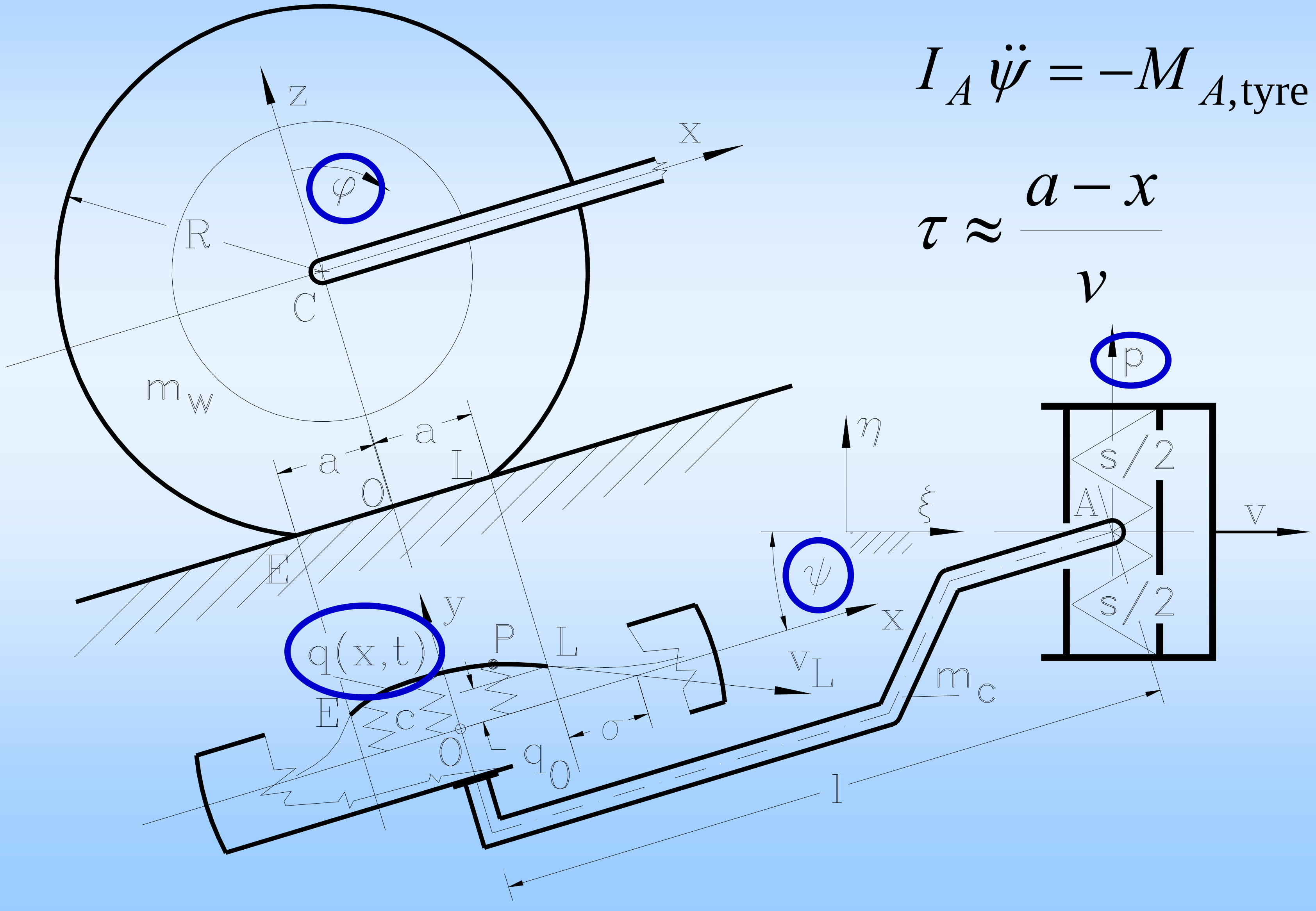
Buses, trucks

video

Mechanical model

$$I_A \ddot{\psi} = -M_{A,\text{tyre}}$$

$$\tau \approx \frac{a - x}{v}$$



Governing equations & memory effect

$$I_A\ddot{\psi}(t) = -c\int_{-a}^{a}(l-x)q(x,t)\mathrm{d}x \qquad p \equiv 0$$

$$\dot{q}(x,t) = v\psi(t) + (l-x)\dot{\psi}(t) + q'(x,t)v + \text{h.o.t.}$$

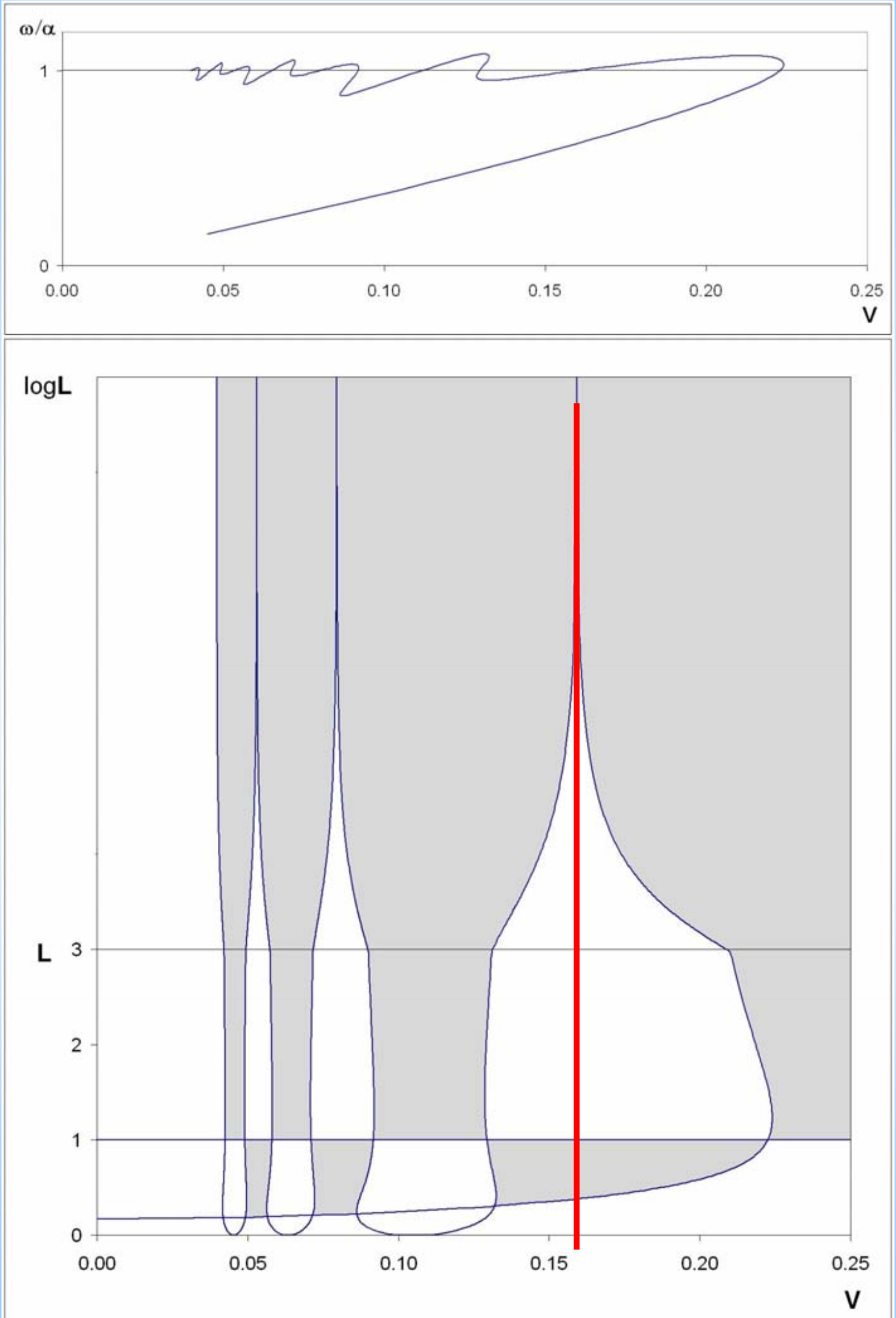
$$x \in [-a,a], \quad t \in [t_0,\infty), \quad \text{and} \quad q(a,t) = 0$$

Traveling wave solution of the PDE:

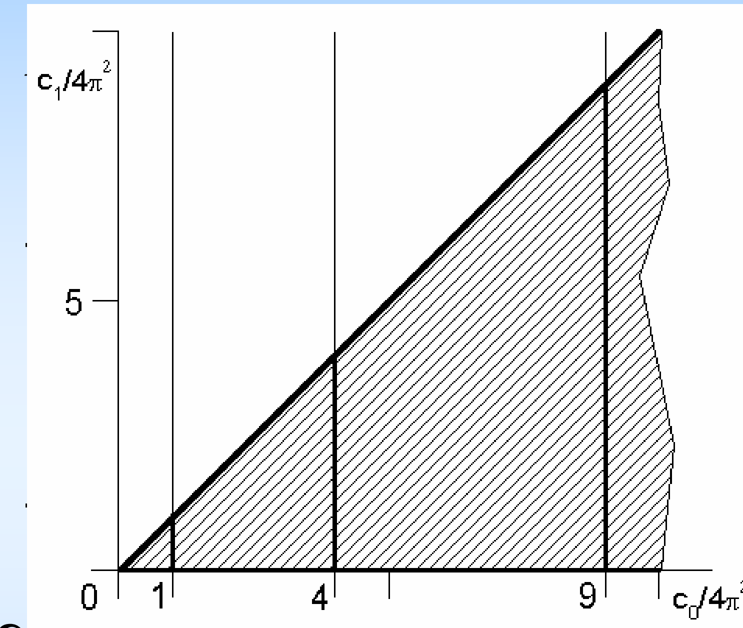
$$q(x,t) = (a-x)\psi(t) + (l-a)(\psi(t) - \psi(t - \underbrace{\tfrac{a-x}{v}}_{=\tau})) + \ldots$$

$$V^2\ddot{\psi}(t) + \psi(t) = \frac{L-1}{L^2+1/3}\int_{-1}^{0}(L-1-2\vartheta)\psi(t+\vartheta)\mathrm{d}\vartheta + \ldots$$

$$V = \frac{v}{2a\omega_n}, \quad L = \frac{l}{a}, \quad \omega_n = \frac{2ac(l^2+a^2/3)}{I_A}$$



$$\ddot{x}(t) + c_0 x(t) = c_1 \int_{-1}^{0} x(t+\vartheta)\,\mathrm{d}\vartheta$$



$$L \to \infty$$

$$V^2 \ddot{\psi}(t) + \psi(t) = \int_{-1}^{0} \psi(t+\vartheta)\,\mathrm{d}\vartheta$$

$$V \neq \frac{1}{2\pi} \approx 0.159, \frac{1}{4\pi}, \frac{1}{6\pi}, \ldots$$

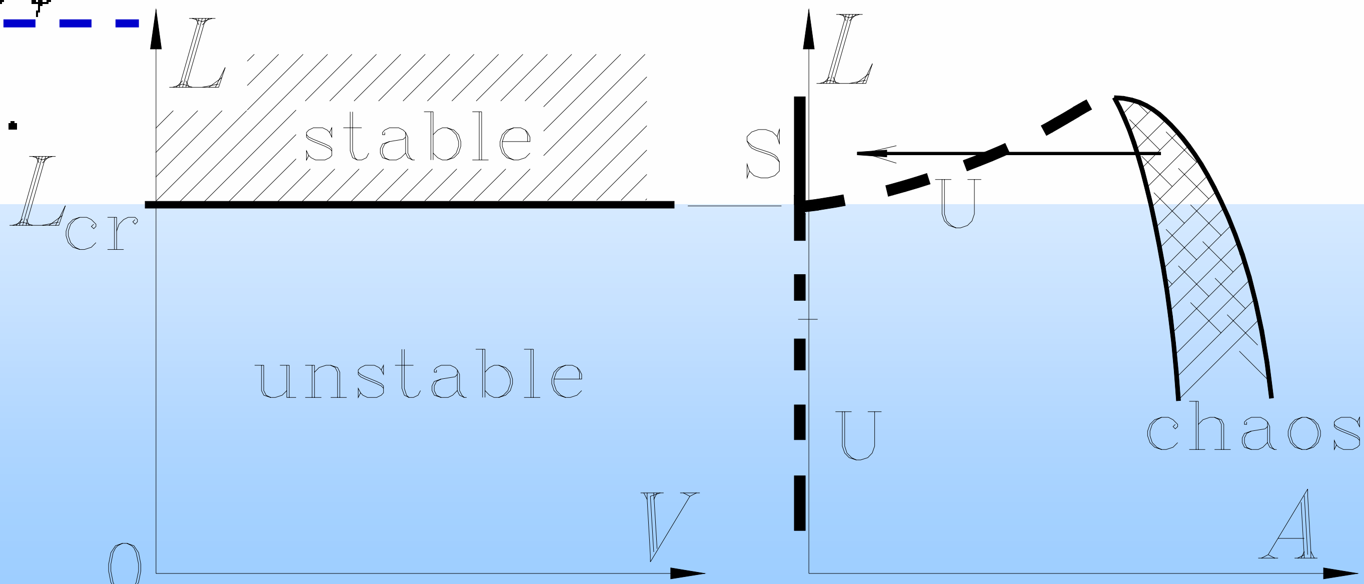
Nonlinear vibrations – without delay

$$\dot{\psi} = \Omega\,,$$

$$\dot{\Omega} = -\frac{\frac{v}{l}\left(\frac{1}{\cos^2\psi} - \frac{1}{2} + \frac{3m_w}{2m_c}\tan^2\psi\right)\Omega + \frac{s}{lm_c}p + \left(1 + \frac{3m_w}{2m_c}\right)\frac{\tan\psi}{\cos\psi}\Omega^2}{\left(\frac{1}{3} + \tan^2\psi\right)\cos\psi + \frac{m_w}{4m_c}\left(\frac{R^2}{l^2}\cos\psi + 6\tan^2\psi\cos\psi\right)}$$

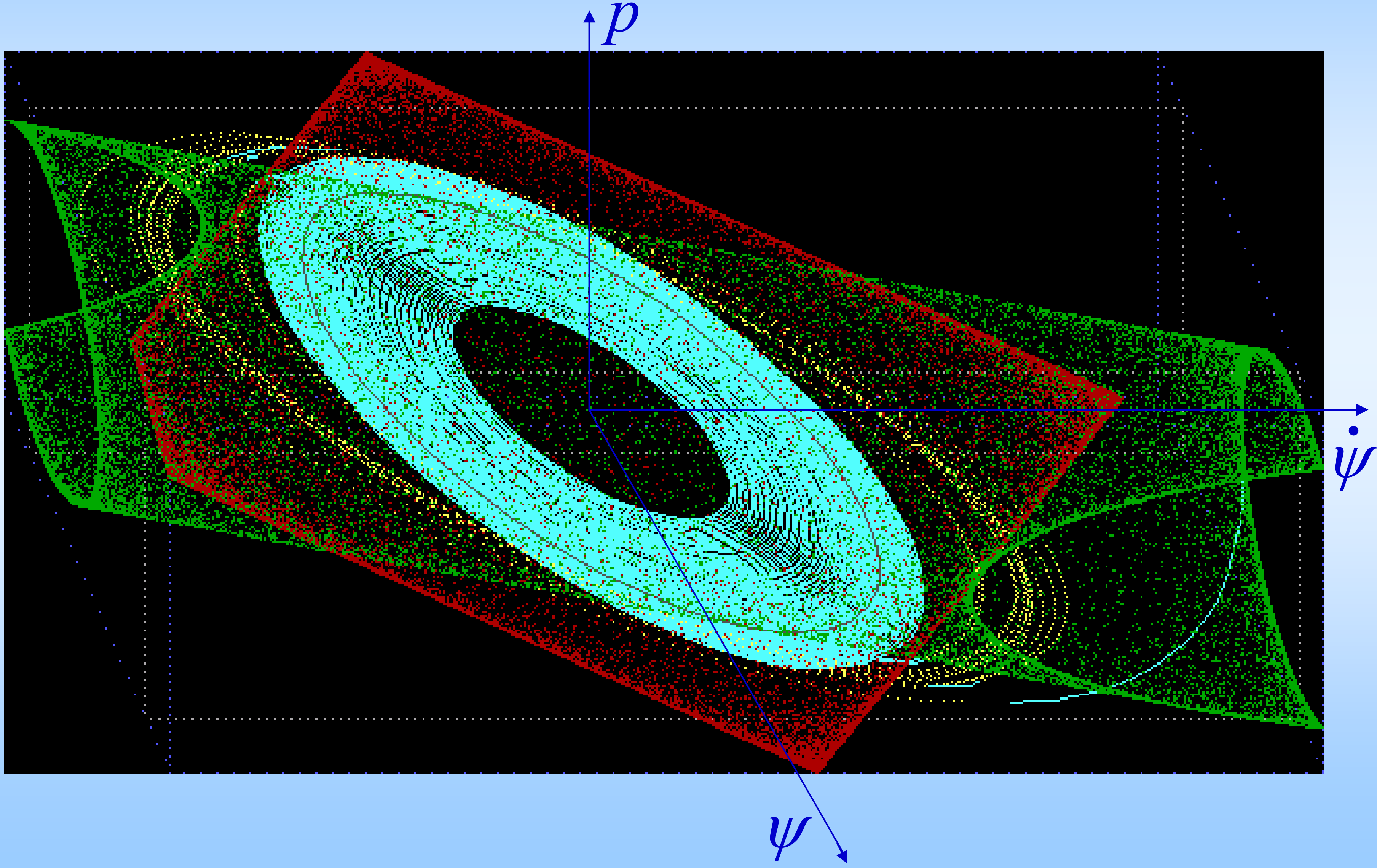
$$\dot{p} = v\tan\psi + \frac{\Omega l}{\cos\psi}\,,$$

$$\dot{\varphi} = \frac{v + \Omega l\sin\psi}{R\cos\psi}\,.$$

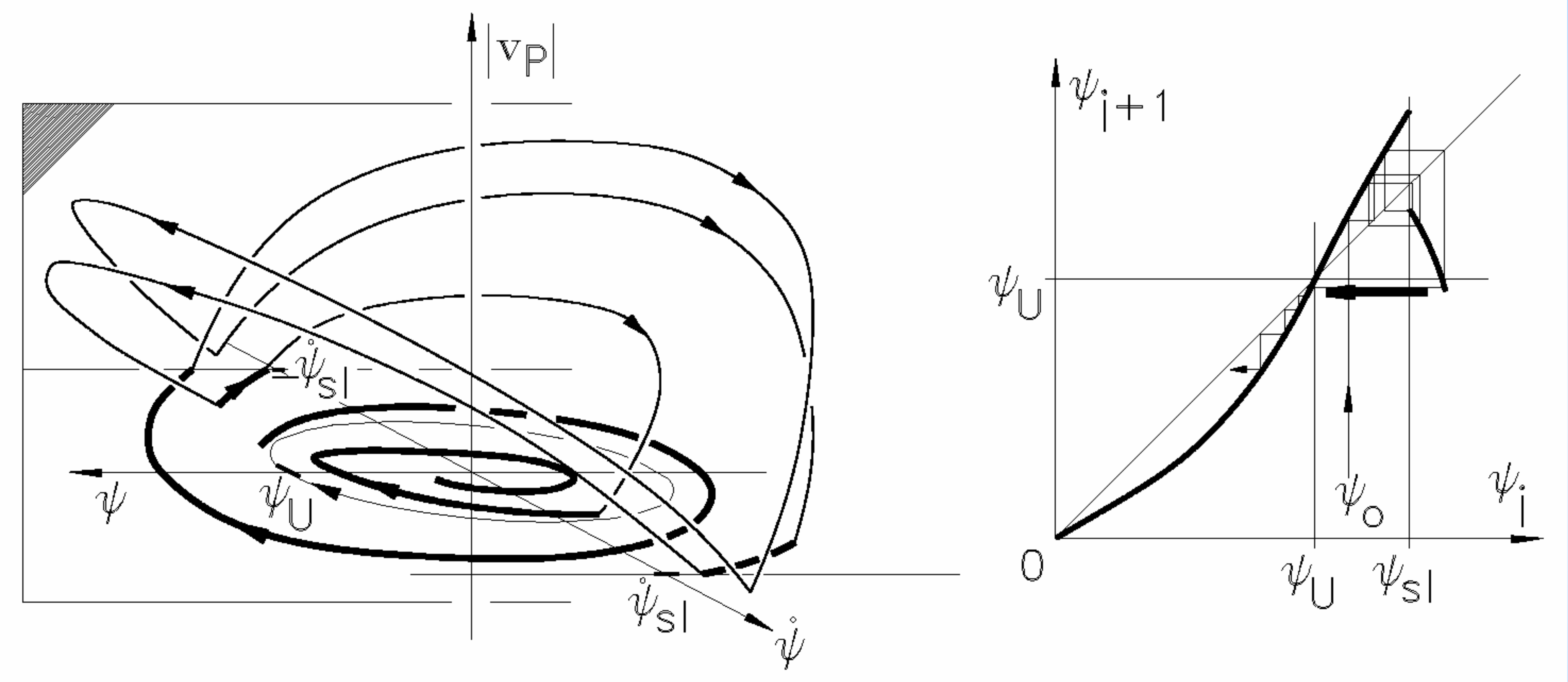


No viscous damping

Transient chaos



Transient chaos



Non-conservative system –

– even without viscous damping!

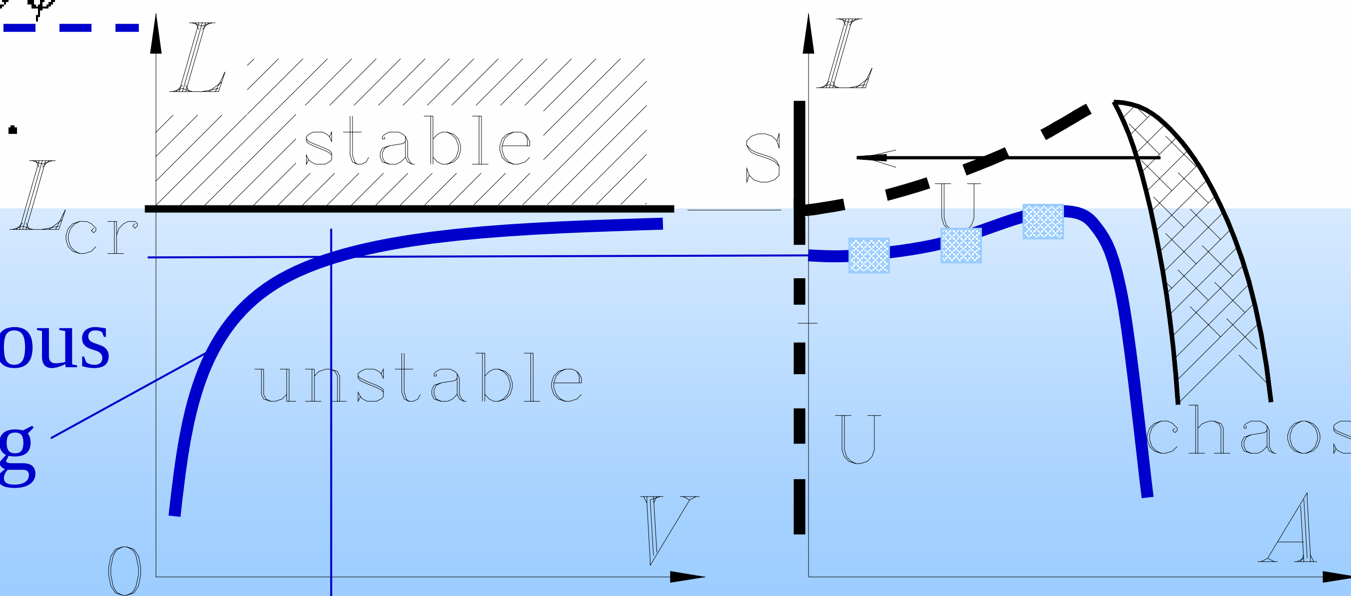
Nonlinear vibrations – without delay

$$\dot\psi = \Omega\,,$$

$$\dot\Omega = -\frac{\frac{v}{l}\left(\frac{1}{\cos^2\psi} - \frac{1}{2} + \frac{3m_w}{2m_c}\tan^2\psi\right)\Omega + \frac{s}{lm_c}p + \left(1+\frac{3m_w}{2m_c}\right)\frac{\tan\psi}{\cos\psi}\Omega^2}{\left(\frac{1}{3}+\tan^2\psi\right)\cos\psi + \frac{m_w}{4m_c}\left(\frac{R^2}{l^2}\cos\psi + 6\tan^2\psi\cos\psi\right)}$$

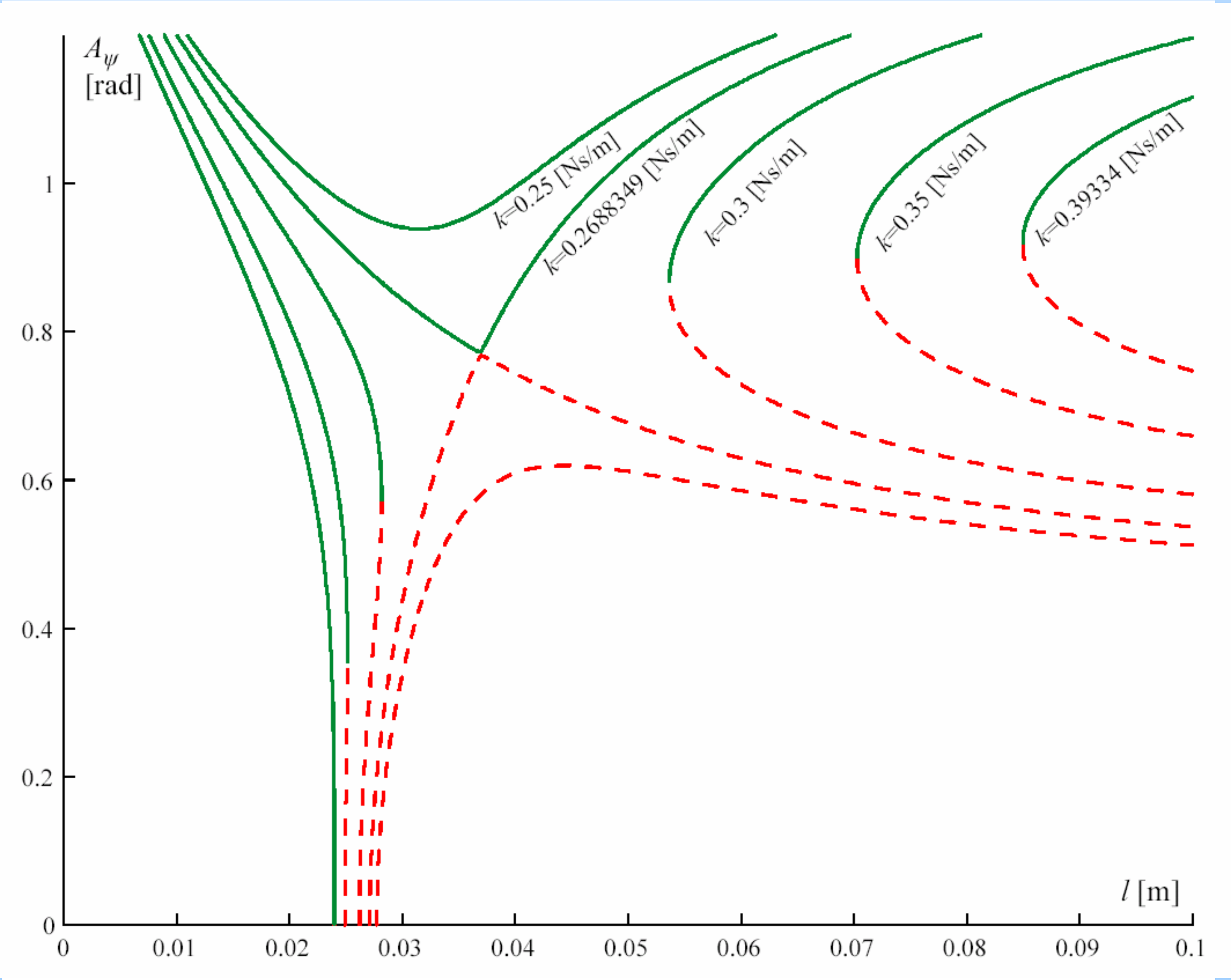
$$\dot p = v\tan\psi + \frac{\Omega l}{\cos\psi}\,,$$

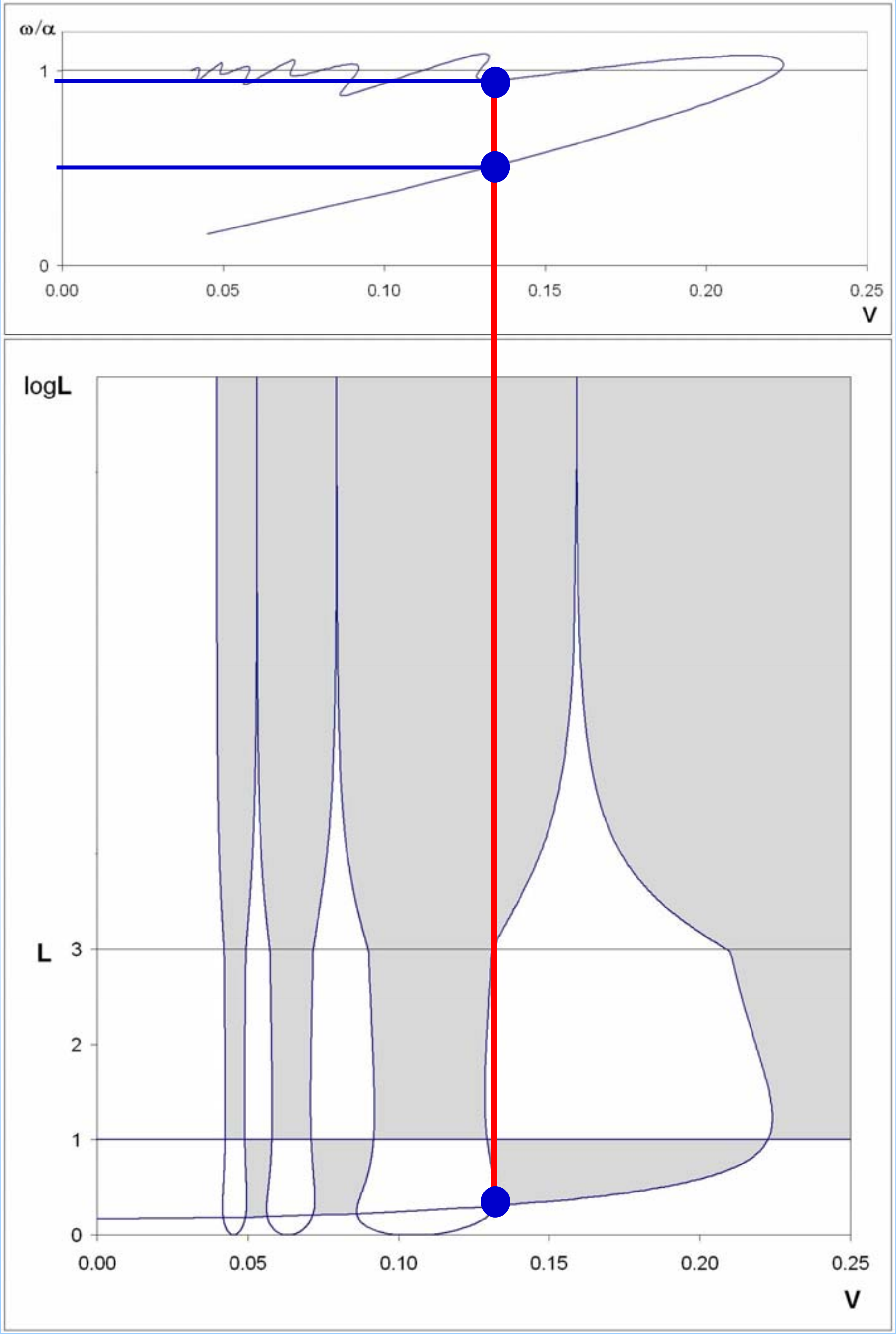
$$\dot\varphi = \frac{v + \Omega l\sin\psi}{R\cos\psi}\,.$$



With viscous damping

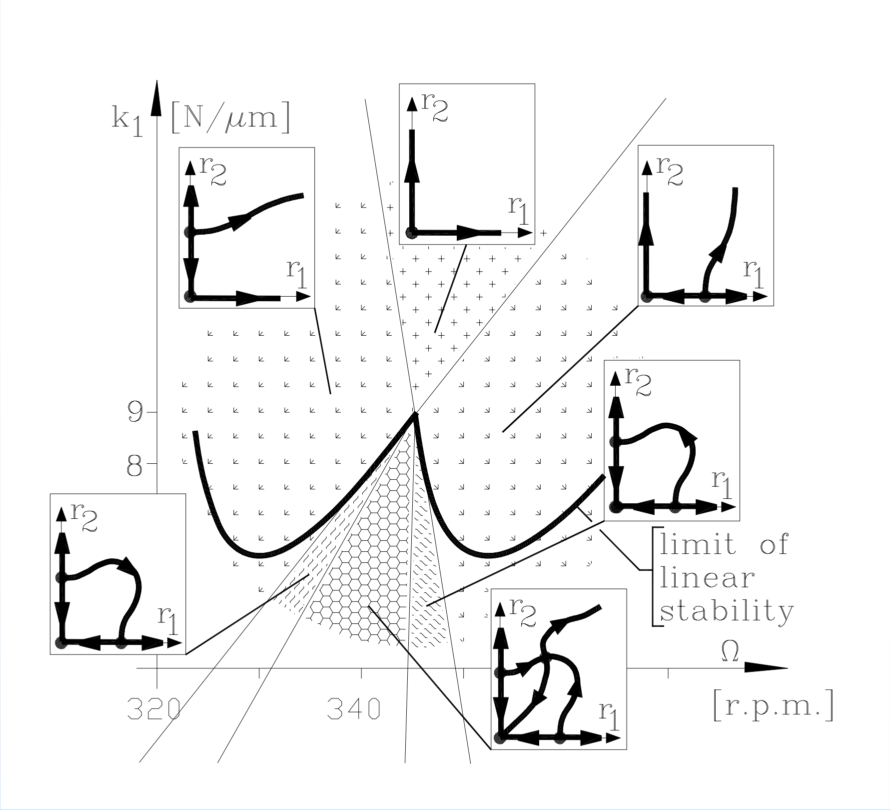
Isola bifurcation



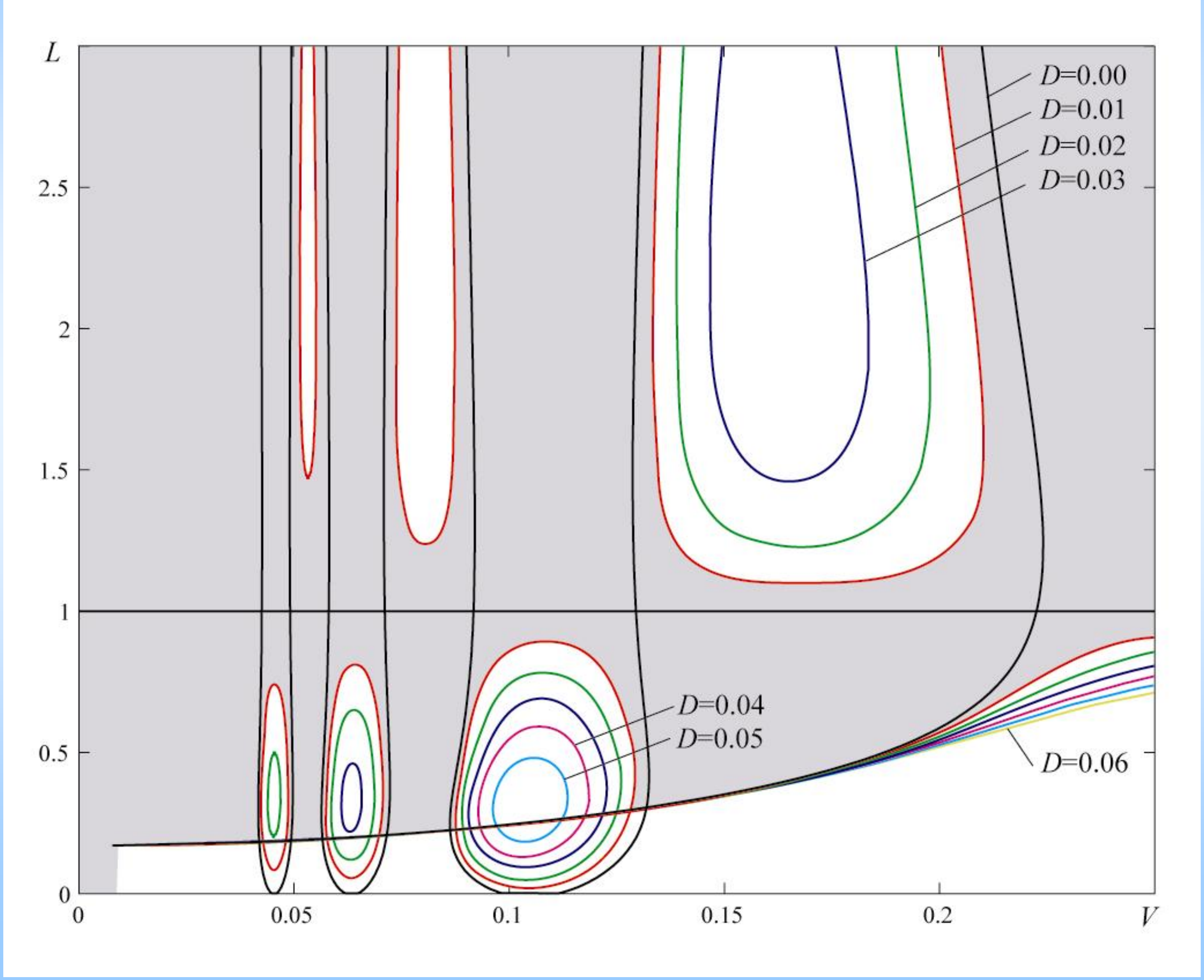


Quasi-periodicity

Subcriticality, like …

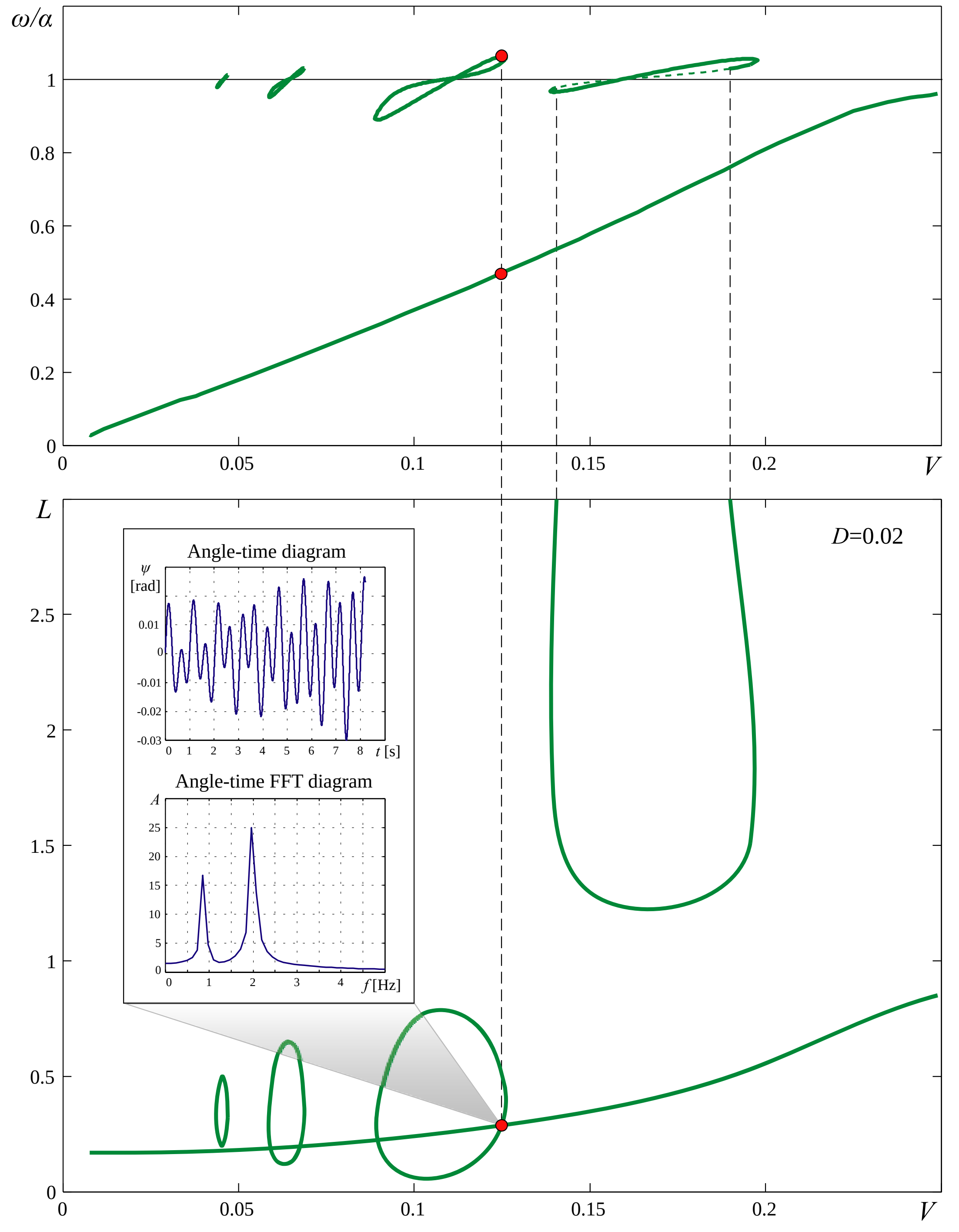


Stability chart with increasing damping

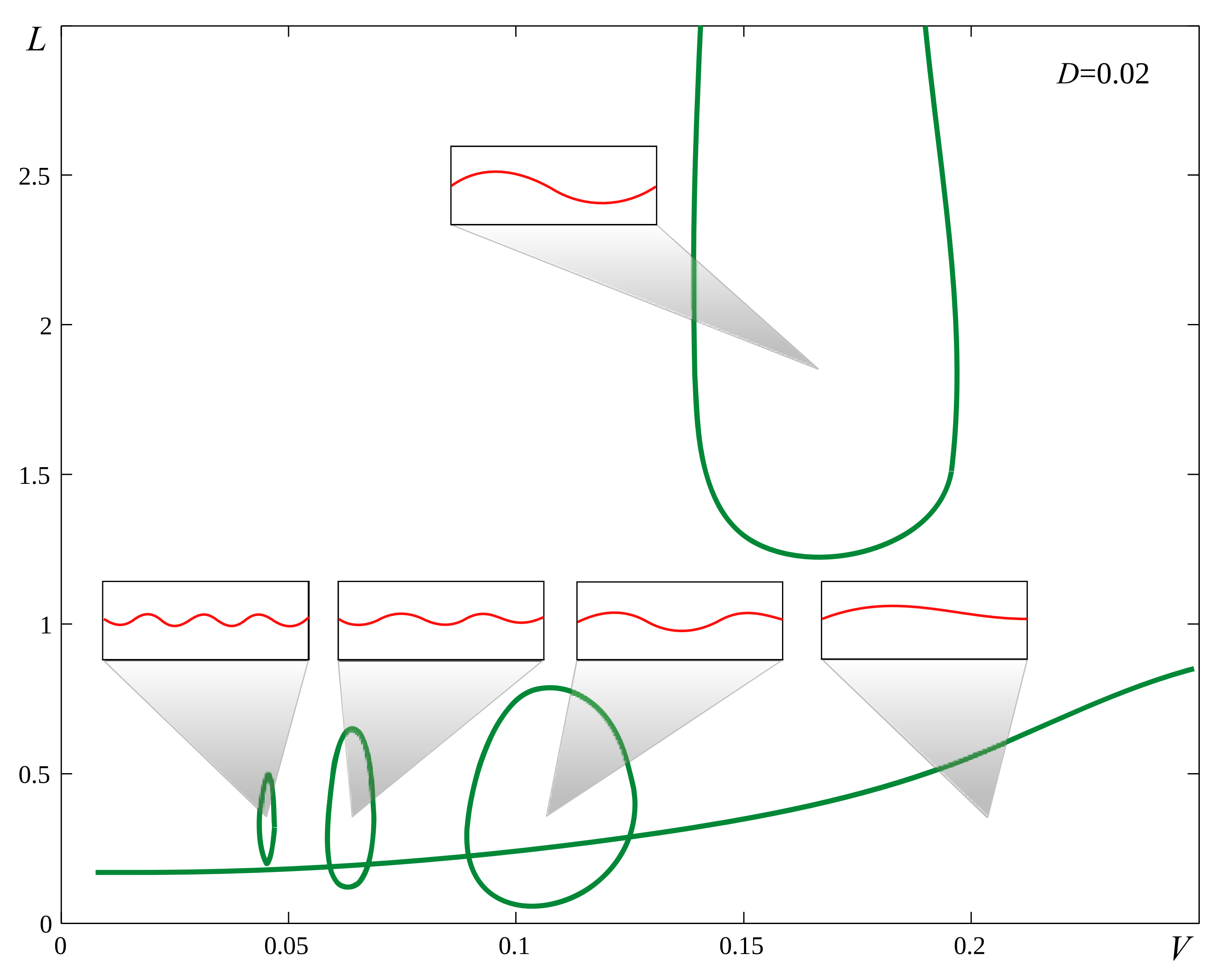


Simulations

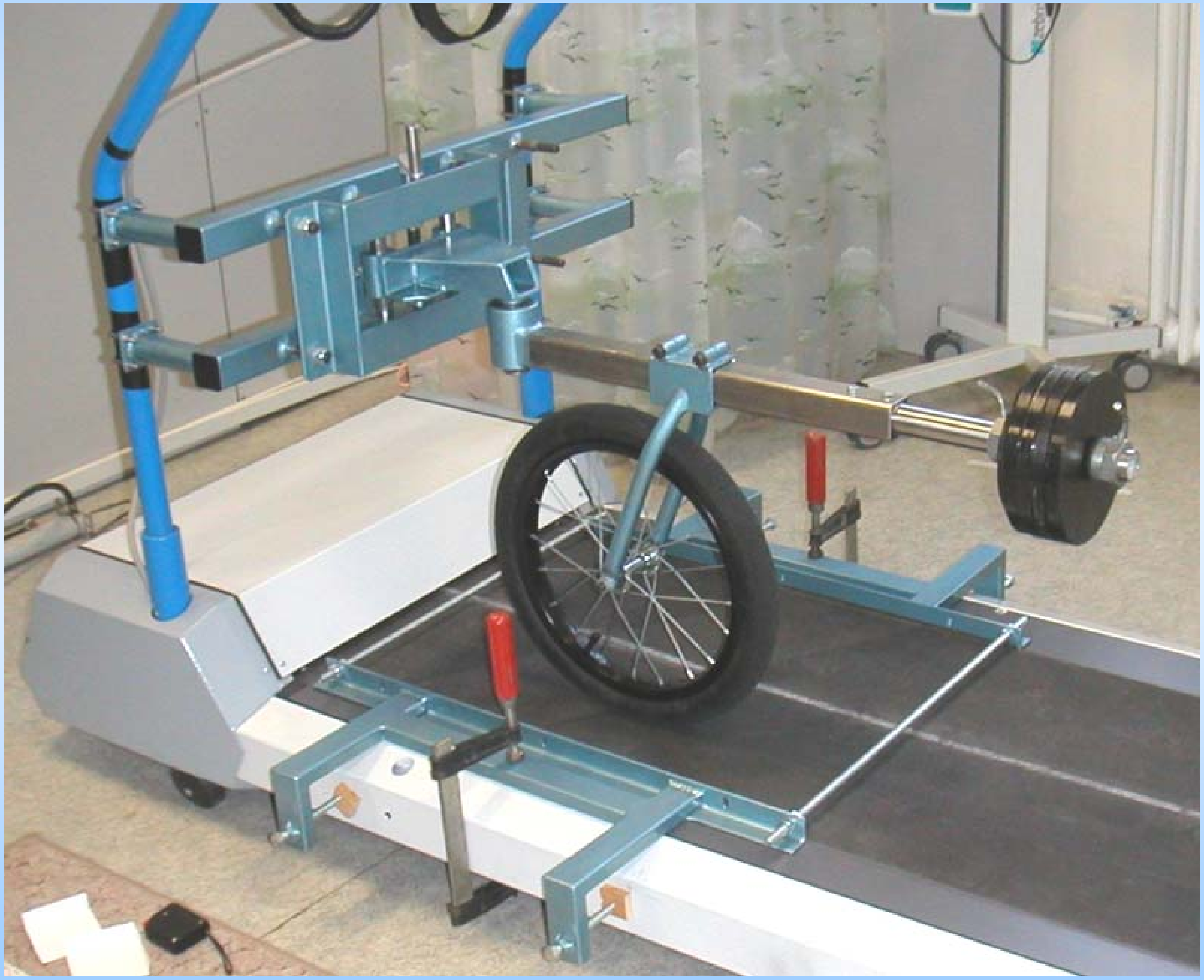
to find quasi-periodic oscillations

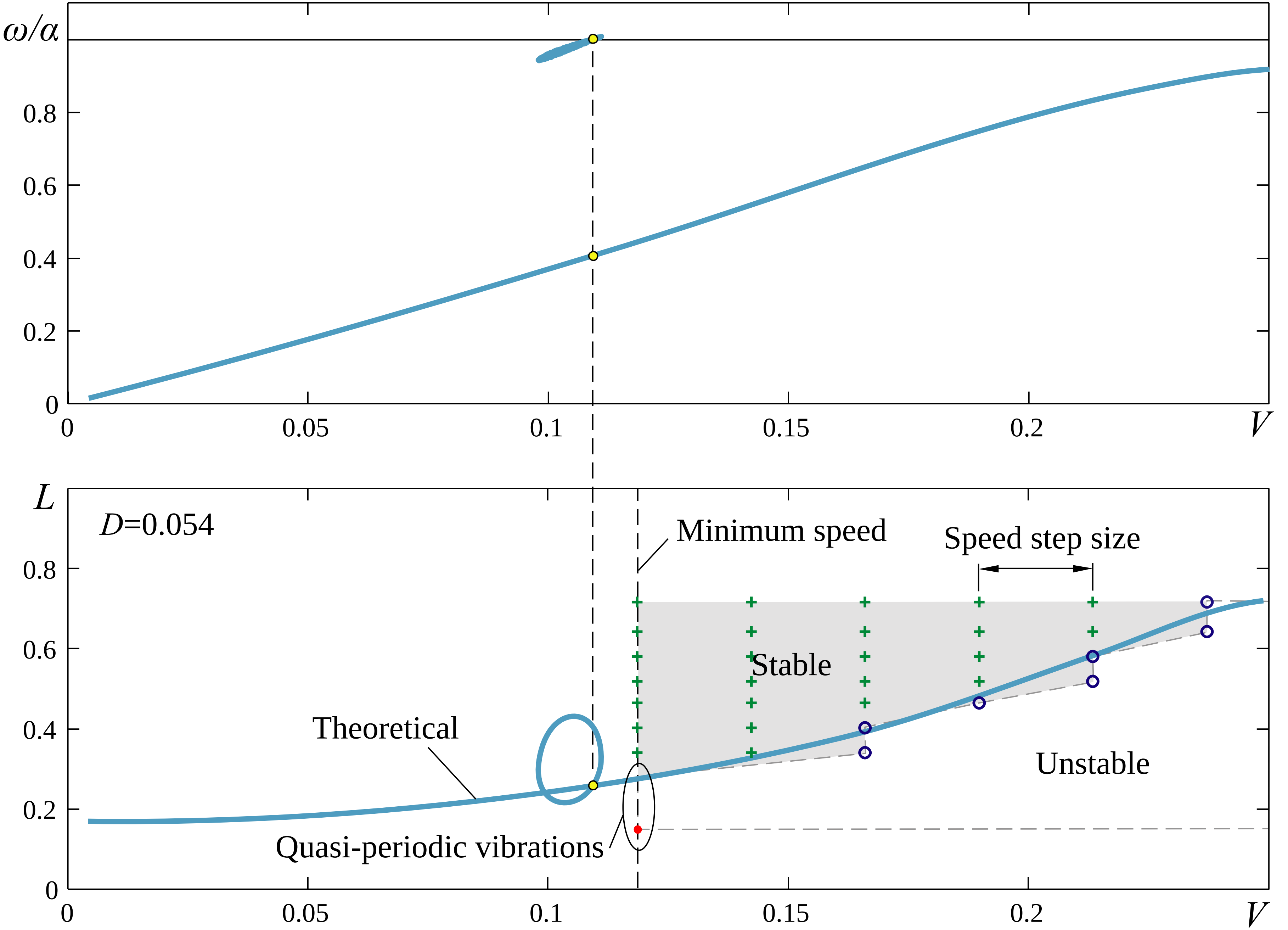


Simulation

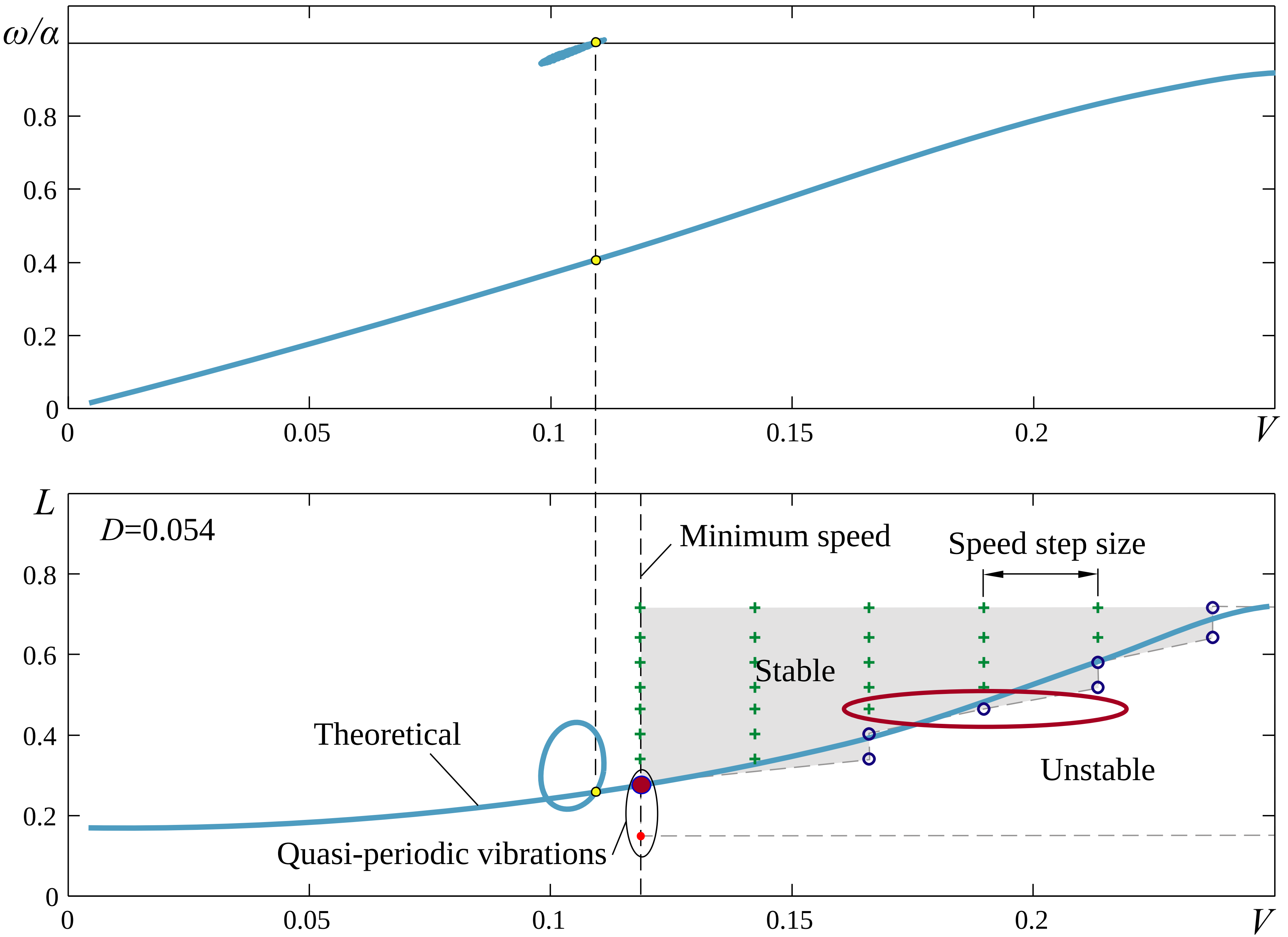


Experiments on running belt





ω/α
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
V
L
D=0.054
Minimum speed
Speed step size
Stable
Theoretical
Unstable
Quasi-periodic vibrations
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
V



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0.2
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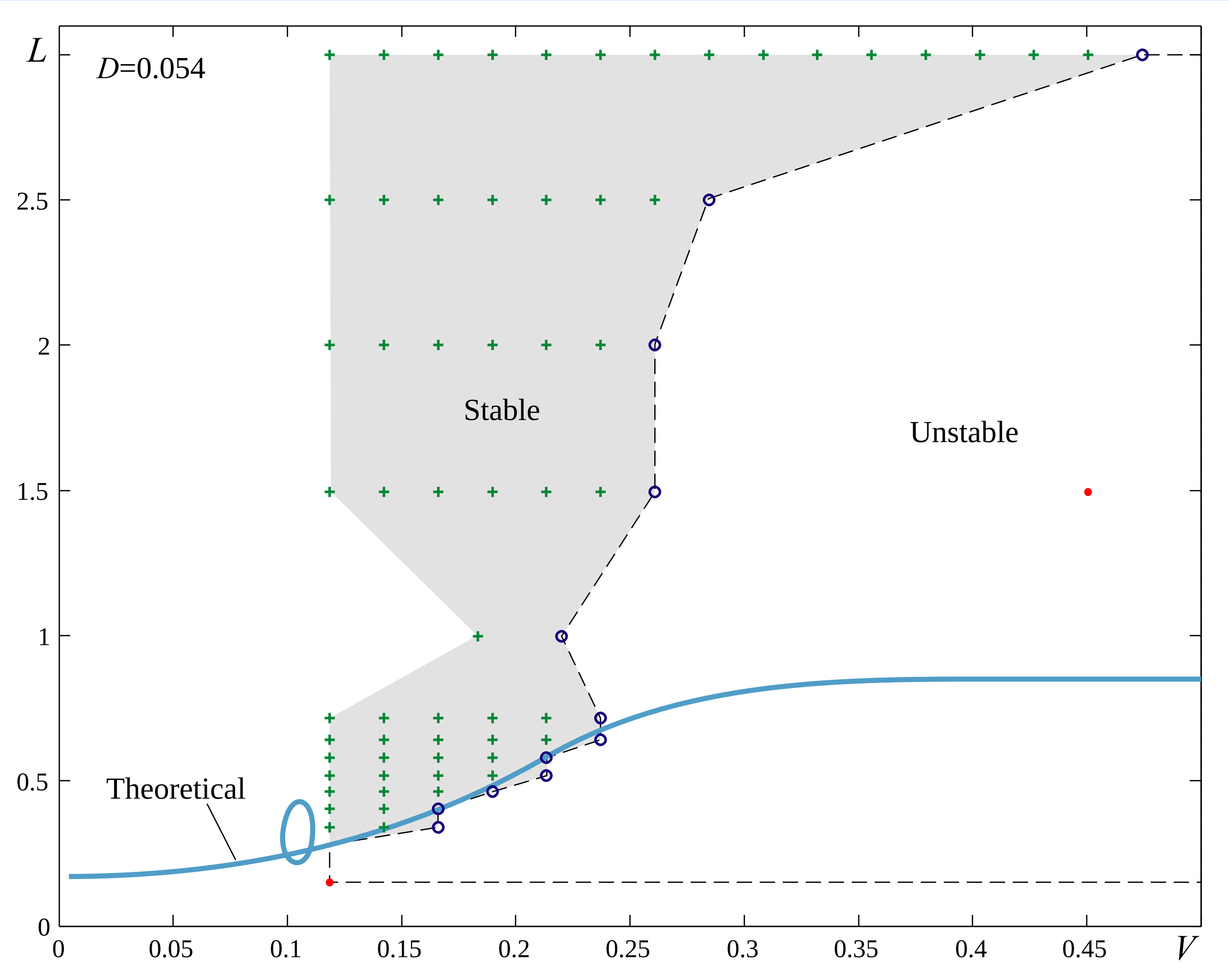
0
0.05
0.1
0.15
0.2
V

Large caster length – elastic belt, king-pin



Conclusion

The quasi-periodic oscillations can be explained with nonlinear time-delay models