A Scale-Normalized Law for Accelerated Odd Collatz Stopping Times Conditioned on a 2-Adic Orbit Statistic

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This is a draft manuscript that the author has paused. If you are interested in continuing this research and publishing a paper, please email a collaboration proposal to:

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Abstract

We report an empirical regularity in the Collatz dynamics observed on integers up to 10^7 (via sampling in large decades). We work with the accelerated map on odd integers

$$T(x) = \frac{3x+1}{2^{v_2(3x+1)}}, \qquad x \text{ odd},$$

and define $s(n)$ as the number of iterations of T needed to reach 1 starting from the odd part of n. For each orbit we also record the average 2-adic valuation

$$\bar{k}(n) = \frac{1}{s(n)} \sum_{j=0}^{s(n)-1} v_2\big(3T^{(j)}(x)+1\big),$$

where x is the odd part of n. We study the scale-normalized quantity $s(n)/\ln n$ and find that, after conditioning on $\bar{k}$, the conditional mean obeys a remarkably stable hyperbolic law across decades:

$$\mathbb{E}\left[\frac{s(n)}{\ln n}\,\middle|\,\bar{k}\right] \approx C + \frac{A}{\bar{k}-b}.$$

The fitted parameters vary slowly with scale and appear to converge toward $C \to 0$, $A \approx 1.40$, $b \approx 1.58$ for large n. This provides a compact, predictive summary of how the orbit-level statistic $\bar{k}$ controls the effective "time constant" in the accelerated odd dynamics.

1 Introduction

1.1 The Collatz map

The Collatz (or $3n+1$) problem studies the iteration on positive integers

$$C(n) = \begin{cases} n/2, & n \text{ even}, \\ 3n+1, & n \text{ odd}. \end{cases} \tag{1}$$

Starting from $n \geq 1$, one forms the orbit $n, C(n), C^{(2)}(n), \ldots$. The classical conjecture asserts that every orbit eventually reaches 1. Despite extensive computational verification and many partial results, a general proof remains unknown.

A key feature of (1) is the alternation between multiplicative expansion (the map $n \mapsto 3n+1$ on odd integers) and contraction by powers of 2 (repeated halving steps on even integers). In particular, if x is odd then $3x+1$ is always even, and often divisible by a high power of 2. This motivates separating the "odd events" from the subsequent runs of divisions by 2.

1.2 The 2-adic valuation

For an integer $m \geq 1$, let $v_2(m)$ denote the exponent of 2 in its prime factorization:

$$v_2(m) = \max\{e \geq 0 : 2^e \mid m\}.$$

Thus $v_2(m) = 0$ iff m is odd, and if m is even then $m = 2^{v_2(m)} \cdot m_{\text{odd}}$ with m_{odd} odd.

In the Collatz setting, for odd x one has

$$3x + 1 = 2^k\, y, \qquad y \text{ odd}, \qquad k = v_2(3x+1) \geq 1,$$

so a single odd step is naturally followed by k halving steps to return to an odd integer. The values of k act as the main source of variability in how quickly an orbit decreases.

1.3 Accelerated dynamics on odd integers

To expose structure, it is common to compress each block of even steps after an odd step into a single transition on odd integers.

Definition 1 (Accelerated odd Collatz map)**.** *For odd $x \geq 1$, define*

$$T(x) = \frac{3x+1}{2^{v_2(3x+1)}}. \tag{2}$$

Then $T(x)$ is odd, and iterating T follows the Collatz orbit of x while skipping all intermediate even values.

Every positive integer n can be written uniquely as $n = 2^a x$ with x odd; we call x the *odd part* of n. Since C simply halves powers of 2, the long-term behavior of the orbit of n is quickly inherited from the orbit of its odd part. This motivates measuring "time to reach 1" using the number of odd-to-odd transitions under T.

2 Definitions and measured quantities

2.1 Odd-step stopping time

Definition 2 (Odd-step stopping time)**.** *Let x be odd. Define*

$$s(x) = \min\{t \geq 0 : T^{(t)}(x) = 1\}.$$

For general $n \geq 1$ with odd part x, define $s(n) = s(x)$.

Thus $s(n)$ counts how many accelerated odd transitions are needed to reach 1 (starting from the odd part of n). This differs from the classical total stopping time, which counts every halving step separately.

2.2 An orbit-level 2-adic statistic

Each accelerated transition $x \mapsto T(x)$ uses the random-looking quantity $k(x) = v_2(3x+1)$. We aggregate these along the orbit.

Definition 3 (Average 2-adic valuation along the odd orbit)**.** *For odd x with $s(x) \geq 1$, define*

$$\bar{k}(x) = \frac{1}{s(x)} \sum_{j=0}^{s(x)-1} v_2\big(3T^{(j)}(x) + 1\big). \tag{3}$$

For general $n \geq 2$ with odd part x, set $\bar{k}(n) = \bar{k}(x)$.

This statistic captures how strongly the orbit tends to contract after odd events: larger $\bar{k}$ means that on average $3x + 1$ contains more factors of 2, leading to larger odd-to-odd contraction.

2.3 Logarithmic normalization

Many heuristics model Collatz as a random walk in $\log n$. Along an odd-to-odd step $x \mapsto T(x)$, ignoring the $+1$,

$$T(x) \approx \frac{3x}{2^{k(x)}} \quad \Rightarrow \quad \Delta \log x \approx \log 3 - k(x) \log 2.$$

Averaging suggests that the number of odd steps to reduce $\log n$ by $\log n$ should scale like a constant multiple of $\log n$. We therefore study the normalized quantity

$$R(n) = \frac{s(n)}{\ln n}, \qquad n \geq 2. \tag{4}$$

3 Empirical finding: a hyperbolic conditional law

We computed $s(n)$ and $\bar{k}(n)$ for n up to 10^6 exactly, and extended the scale comparison to 10^7 via uniform sampling within decades (see §4). We then estimated the conditional mean

$$f(\bar{k}) = \mathbb{E}\big[R(n) \mid \bar{k}(n) = \bar{k}\big]$$

by binning $\bar{k}$ values and averaging $R(n)$ within each bin.

3.1 A stable hyperbola

Across multiple decades, the binned conditional mean is fit extremely well by a simple hyperbolic model

$$f(\bar{k}) \approx C + \frac{A}{\bar{k} - b}, \tag{5}$$

with parameters that vary slowly with scale and appear to stabilize for large n.

For example, using quantile bins (to keep bin counts comparable) and weighted least squares on the binned points, we obtained the following fits:

Decade	b	A	C	R^2
$[10^3, 10^4)$	1.544	1.649	−0.240	0.998
$[10^4, 10^5)$	1.562	1.522	−0.121	1.000
$[10^5, 10^6)$	1.578	1.425	−0.030	1.000
$[10^6, 10^7)$	1.584	1.398	0.001	1.000

These numbers suggest a limiting behavior (at least on the observed range)

$$C \to 0, \qquad A \approx 1.40, \qquad b \approx 1.58,$$

so that for large scale one may heuristically write

$$\mathbb{E}\left[\frac{s(n)}{\ln n}\,\middle|\,\bar{k}\right] \approx \frac{A_\infty}{\bar{k} - b_\infty} \quad \text{with} \quad A_\infty \approx 1.40, \quad b_\infty \approx 1.58. \tag{6}$$

3.2 Interpretation in log-drift terms

Since an odd-to-odd step has approximate log-change

$$\Delta \log x \approx \log 3 - k(x) \log 2,$$

the orbit-average drift is approximately

$$\overline{\Delta \log}(n) \approx \log 3 - \bar{k}(n) \log 2.$$

Thus $\bar{k}$ directly controls the typical speed at which an orbit decreases on the log scale. The empirical law (5) can be read as an effective "time constant" relationship: orbits with smaller $\bar{k}$ (less division by 2 on average) have larger normalized time $s(n)/\ln n$, while orbits with larger $\bar{k}$ collapse more quickly.

A natural first-principles guess is that a time scale should behave like the inverse of the magnitude of the drift, i.e. proportional to $1/(\bar{k} \ln 2 - \ln 3)$. In practice, we found that the fitted hyperbola in $\bar{k}$ with a shift b provides an even tighter description on the observed range, absorbing discretization and finite-scale effects.

4 Methods

4.1 Computation of $s(n)$ and $\bar{k}(n)$

We computed $s(n)$ and the sum of $k(x) = v_2(3x+1)$ along the odd orbit by memoization on odd integers. For each odd x encountered, we store both $s(x)$ and $\sum k$ from x to 1, allowing rapid evaluation for many starting values. For a general n, we reduce to its odd part before applying the memoized values.

4.2 Conditioning and binning

To estimate $f(\bar{k})$, we binned $\bar{k}$ values within each decade using quantile bins (equal-count bins), and computed the mean of $R(n) = s(n)/\ln n$ in each bin. Fits to (5) were performed by weighted least squares using bin counts as weights.

For the decade $[10^6, 10^7)$ we used uniform sampling of 200,000 integers to control runtime while retaining stable bin statistics.

5 Discussion and outlook

The main empirical observation is that, after logarithmic normalization and conditioning on the orbit statistic $\bar{k}$, the accelerated odd stopping time exhibits a simple and highly stable hyperbolic law across decades. The fact that the same functional form fits well on $[10^3, 10^4)$ through $[10^6, 10^7)$ suggests that $\bar{k}$ captures a dominant mechanism controlling the effective speed of the odd Collatz dynamics.

Several directions are natural:

- Extend the decade comparison to $[10^7, 10^8)$ and beyond (with sampling), to test convergence of (A, b, C).
- Replace $\bar{k}$ by more robust orbit descriptors (e.g. median of k, or frequency of small k values) to see whether the law tightens further.
- Compare the empirical hyperbola to drift-based predictions derived from stochastic models, clarifying how finite-scale and arithmetic effects induce the observed shift parameter b.

Reproducibility. All quantities in this note are directly computable from the definitions above; the experiment consists of evaluating $(s(n), \bar{k}(n))$ for many n, forming $R(n) = s(n)/\ln n$, and fitting (5) to binned conditional means over decade intervals.